HYBRID ARQ IN WIRELESS NETWORKS

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AUTOMATIC REPEAT REQUEST

• The receiving end detects frame errors and requests retransmissions.

• $P_e$ is the frame error rate, the average number of transmissions is

\[1 \cdot (1 - P_e) + \cdots + n \cdot P_e^{n-1} (1 - P_e) + \cdots = \frac{1}{1 - P_e}\]

• Hybrid ARQ uses a code that can correct some frame errors.

• In HARQ schemes
  – the average number of transmissions is reduced, but
  – each transmission carries redundant information.
Decoding the name of an information theorist from its noisy version:

EMRE
ARQ
An Example

• Decoding the name of an information theorist from its noisy version:

EMRE
ARQ

An Example

• Decoding the name of an information theorist from its noisy version:

  MRE
ARQ
An Example

• Decoding the name of an information theorist from its noisy version:

\[\text{IMRE}\]

• Increasing redundancy:

\[\text{EMR E}\]
ARQ

An Example

• Decoding the name of an information theorist from its noisy version:

  E M R E

• Increasing redundancy:

  E M R E T E L A T A R
ARQ
An Example

- Decoding the name of an information theorist from its noisy version:

  IMRE

- Increasing redundancy:

  EMRE TELATAR

  IMRE

  IMRE
ARQ
An Example

• Decoding the name of an information theorist from its noisy version:

   E M R E
   I M R E

• Increasing redundancy:

   E M R E T E L A T A R
   I M R E C S I S Z A R
THROUGHPUT IN HYBRID ARQ
BPSK, AWGN, BCH Coded

$E_s/N_0$ [dB]
TYPE II HYBRID ARQ

Incremental Redundancy

- Puncturing:

   E M R E T E L A T A R
TYPE II HYBRID ARQ
Incremental Redundancy

• Puncturing:

E M R E T E L A T A R

• Rate compatible:

M R E
TYPE II HYBRID ARQ
Incremental Redundancy

• Puncturing:

\[ \text{E M R E T E L A T A R} \]

• Rate compatible:

\[ \text{M R E A R} \]
TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

  E M R E T E L A T A R

- Rate compatible:

  M R E T E L A A R
TYPE II HYBRID ARQ
Incremental Redundancy

• Puncturing:

E M R E T E L A T A R

• Rate compatible:

E M R E T E L A T A R
TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:
  EMRETELATAR

- Rate compatible:
  EMRETELATAR

- Not rate compatible:
  MRE
TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

  E M R E T E L A T A R

- Rate compatible:

  E M R E T E L A T A R

- Not rate compatible:

  M E A R
TYPE II HYBRID ARQ
Incremental Redundancy

• Puncturing:

    E M R E T E L A T A R

• Rate compatible:

    E M R E T E L A T A R

• Not rate compatible:

    M E T E L A A
TYPE II HYBRID ARQ
Incremental Redundancy

• Puncturing:

   E M R E T E L A T A R

• Rate compatible:

   E M R E T E L A T A R

• Not rate compatible:

   E E T E L T A
TYPE II HYBRID ARQ
Incremental Redundancy

- Puncturing:

  E M R E T E L A T A R

- Rate compatible:

  E M R E T E L A T A R

- Not rate compatible:

  E E T E L T A
TYPE II HYBRID ARQ
Incremental Redundancy

- Information bits are encoded by a (low rate) mother code.
- Information and a selected number of parity bits are transmitted.
- If a retransmission is not successful:
  - transmitter sends additional selected parity bits
  - receiver puts together the new bits and those previously received.
- Each retransmission produces a codeword of a stronger code.
- Family of codes obtained by puncturing of the mother code.
INCREMENTAL REDUNDANCY

A Rate $\frac{1}{5}$ Mother Code

\[ \text{at the transmitter} \]
INCREMENTAL REDUNDANCY

A Rate 1/5 Mother Code

---

at the transmitter

transmission #1

---

at the receiver
INCREMENTAL REDUNDANCY

A Rate 1/5 Mother Code

---

at the transmitter

transmission # 1

transmission # 2

---

at the receiver
INCREMENTAL REDUNDANCY

A Rate $\frac{1}{5}$ Mother Code

At the transmitter

Transmission # 1

Transmission # 2

Transmission # 3

At the receiver
INCREMENTAL REDUNDANCY

A Rate $1/5$ Mother Code

at the transmitter

transmission # 1

transmission # 2

transmission # 3

transmission # 4

at the receiver
THROUGHPUT IN HYBRID ARQ

HARQ Scheme based on Turbo codes in AWGN Channel

Throughput of new puncturing scheme
Throughput of standard
BPSK Capacity
Cutoff Rate
RANDOMLY PUNCTURED CODES

- The mother code is an \((n, k)\) rate \(R\) turbo code.
- Each bit is punctured independently with probability \(\lambda\).
- The expected rate of the punctured code is \(R/(1 - \lambda)\).
- For large \(n\) we have

\[
\begin{align*}
\text{TURBO CODE} & \quad \text{PUNCTURING DEVICE} \\
k \text{ BITS} & \quad n \text{ BITS} & \quad (1 - \lambda)n \text{ BITS}
\end{align*}
\]
A FAMILY OF RANDOMLY PUNCTURED CODES

Rate Compatible Puncturing

• The mother code is an \((n, k)\) rate \(R\) turbo code.

• \(\lambda_j\) for \(j = 1, 2, \ldots, m\) are puncturing rates, \(\lambda_j > \lambda_k\) for \(j < k\).

• If the \(i\)-th bit is punctured in the \(k\)-th code and \(j < k\), then it was punctured in the \(j\)-th code.

• \(\theta_i\) for \(i = 1, 2, \ldots, n\) are uniformly distributed over \([0, 1]\).

• If \(\theta_i < \lambda_l\), then the \(i\)-th bit is punctured in the \(l\)-th code.
MEMORYLESS CHANNEL MODEL

- Binary input alphabet \( \{0, 1\} \) and output alphabet \( \mathcal{Y} \).

- Constant in time with transition probabilities \( W(b|0) \) and \( W(b|1) \), \( b \in \mathcal{Y} \).

- Time varying with transition probabilities at time \( i \) \( W_i(b|0) \) and \( W_i(b|1) \), \( b \in \mathcal{Y} \).

- \( W_i(\cdot|0) \) and \( W_i(\cdot|1) \) are known at the receiver.
PERFORMANCE MEASURE

Time Invariant Channel

- Sequence $x \in C \subseteq \{0, 1\}^n$ is transmitted, and $x'$ decoded.

- Sequences $x$ and $x'$ are at Hamming distance $d$.

- The probability of error $P_e(x, x')$ can be bounded as

$$P_e(x, x') \leq \gamma^d = \exp\{-d\alpha\},$$

where $\gamma$ is the Bhattacharyya noise parameter:

$$\gamma = \sum_{b \in \mathcal{Y}} \sqrt{W(b|0)W(b|1)}$$

and $\alpha = -\log \gamma$ is the Bhattacharyya distance.
PERFORMANCE MEASURE

• An \((n, k)\) binary linear code \(C\) with \(A_d\) codewords of weight \(d\).

• The union-Bhattacharyya bound on word error probability:

\[
P_{W}^{C} \leq \sum_{d=1}^{n} A_d e^{-\alpha d}.
\]

• Weight distribution \(A_d\) for a turbo code?

• Consider a set of codes \([C]\) corresponding to all interleavers.

• Use the average \(\overline{A}_d^{[C]}(n)\) instead of \(A_d\) for large \(n\).
• There is an ensemble distance parameter $c^C_0$ s.t. for large $n$,

$$\overline{A}_d^{[C]}(n) \leq \exp(d c^C_0) \text{ for large enough } d.$$ 

• For a channel whose Bhattacharyya distance $\alpha > c^C_0$, we have

$$\overline{P}_W^{[C]}(n) = O(n^{-\beta}).$$

• $c^C_0$ is the ensemble noise threshold.
• Is there the punctured ensemble noise threshold $c_{0}^{[C_P]}$:

$$\overline{A}_{j}^{[C_P]}(n) \leq \exp(jc_{0}^{[C_P]}) \text{ for large enough } n \text{ and } j.$$  

• The expected number of codewords of weight $j$:

$$\overline{A}_{j}^{[C_P]}(n) = \sum_{d \geq j} \overline{A}_{d}^{[C]}(n) \binom{d}{j} \lambda^{d-j}(1 - \lambda)^j$$

• If $\log \lambda < -c_{0}^{[C]}$,

$$c_{0}^{[C_P]} \leq \log \left[ \frac{1 - \lambda}{\exp(-c_{0}^{[C]}) - \lambda} \right].$$
PUNCTURED TURBO CODE ENSEMBLES

Throughput vs. Es/N0 (dB) for different punctured turbo codes and BPSK capacity. The graph shows the cutoff rate for $R=0.82$. The punctured turbo codes are denoted as follows:
- Punctured TC $k=384$
- Punctured TC $k=3840$
- BPSK Capacity
- Cutoff Rate
HARQ MODEL

• There are at most $m$ transmissions.

• $I = \{1, \ldots, n\}$ is the set indexing the bit positions in a codeword.

• $I$ is partitioned in $m$ subsets $I(j)$, for $1 \leq j \leq m$.

• Bits at positions in $I(j)$ are transmitted during $j$-th transmission.

• The channel remains constant during a single transmission:

$$\gamma_i = \gamma(j) \text{ for all } i \in I(j).$$
PERFORMANCE MEASURE

Time Varying Channel

• Let $W^n(y|x) = \prod_{i=1}^{n} W_i(y_i|x_i)$.

• Sequence $x \in C \subseteq \{0, 1\}^n$ is transmitted, and $x'$ decoded.

• The probability of error $P_e(x, x')$ can be bounded as

$$P_e(x, x') \leq \sum_{y \in \mathcal{Y}^n} \sqrt{W^n(y|x)W^n(y|x')}$$

$$= \prod_{i=1}^{n} \left( \sum_{b \in \mathcal{Y}} \sqrt{W_i(b|x_i)W_i(b|x_i')} \right)$$

$$\leq \prod_{i : x_i \neq x'_i} \gamma_i$$
HARQ PERFORMANCE

• $d_j$ is the Hamming distance between $x$ and $x'$ over $I(j)$.

• The probability of error $P_e(x, x')$ can be bounded as

$$P_e(x, x') \leq \prod_{j=1}^{m} \gamma(j)^{d_j}$$

• $A_{d_1 \ldots d_m}$ is the number of codewords with weight $d_j$ over $I(j)$.

• The union bound on the ML decoder word error probability:

$$P \leq \sum_{d_1=1}^{\lfloor \frac{|I(1)|}{d_1} \rfloor} \cdots \sum_{d_m=1}^{\lfloor \frac{|I(m)|}{d_m} \rfloor} A_{d_1 \ldots d_m} \prod_{j=1}^{m} \gamma(j)^{d_j}$$
HARQ PERFORMANCE
Random Transmission Assignment

• A bit is assigned to transmission \( j \) with probability \( \alpha_j \).

• \( d \) is the weight of the original codeword.

• \( d_j \) is the weight of the \( d \)-th transmission sub-word.

• The probability that the sub-word weights are \( d_1, d_2 \ldots, d_m \) is

\[
{d \choose d_1} {d - d_1 \choose d_2} \cdots {d - d_1 \cdots - d_{m-1} \choose d_m} \alpha_1^{d_1} \alpha_2^{d_2} \cdots \alpha_m^{d_m}
\]
HARQ PERFORMANCE
Random Transmission Assignment

- The union bound on the ML decoder word error probability:

\[ P \leq \sum_{d_1=1}^{\mid I(1)\mid} \cdots \sum_{d_m=1}^{\mid I(m)\mid} A_{d_1 \ldots d_m} \prod_{j=1}^{m} \gamma(j)^{d_j} \]

- The expected value of the union bound is

\[ \sum_{d} A_d \left( \sum_{j=1}^{m} \gamma(j) \alpha_j \right)^d \]

- The average Bhattacharyya noise parameter:

\[ \bar{\gamma} = \sum_{j=1}^{m} \gamma(j) \alpha_j \]
A RANDOMLY PUNCTURED TURBO CODE

An Example of Random Transmission Assignment

- The puncturing probability is $\lambda$.
- Transmission over the channel with noise parameter $\gamma$.
- Equivalent to having two transmissions:
  - first with assignment probability $(1 - \lambda)$ and noise parameter $\gamma$;
  - second with assignment probability $\lambda$ and noise parameter $1$.
- The average noise parameter is $\overline{\gamma} = (1 - \lambda)\gamma + \lambda$.
- The requirement $-\log \overline{\gamma} > c_0^{[C]}$ translates into

$$-\log \gamma > \log \left[ \frac{1 - \lambda}{\exp (\overline{c_0^{[C]}}) - \lambda} \right].$$
INCREMENTAL REDUNDANCY

Concluding Remarks

at the transmitter

- transmission # 1
- transmission # 2
- transmission # 3
- transmission # 4

at the receiver