Price and Service Competition in an Outsourced Supply Chain

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Abstract

We consider a buyer who outsources the manufacturing of a product to multiple symmetric make-to-stock suppliers who compete on price and service (fill rate). The buyer allocates demand to the suppliers using a score function with an exponential form, which specifies the relative importance of price vs. service, in order to minimize his costs, while the suppliers choose their prices and fill rates to maximize their profits. For the case of dual-sourcing, we characterize the optimal parameter of the exponential score function, considering the impact of the buyer’s decisions on the suppliers and considering how the suppliers compete against each other to earn a portion of the buyer’s demand. We prove the existence of a unique equilibrium and characterize the equilibrium behavior of the system. We then consider a general number of suppliers and show that the equilibrium prices and fill rates, and the buyer’s cost, are increasing in the number of suppliers. We compare these results to a model of single-sourcing, in which the buyer is the Stackelberg leader and extracts all profits from the supplier. We find that the buyer always prefers single-sourcing to multi-sourcing. Finally, we study a centralized system and use the results to develop a coordinating contract for the decentralized system.

Keywords: Single-sourcing, Multi-sourcing, Competition, Pricing, Coordination

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1 Introduction and Motivation

In many industries, including telecommunications, electronics and automotive, the past two decades have seen a sharp increase in the use of outsourcing and contract manufacturing, with the goal of reducing costs and obtaining operational efficiencies (McIvor, 2003). Global manufacturers who have chosen to outsource often make the decision to multi-source the production of key products or components in order to mitigate the risk of not being able to satisfy all customer demand, to protect against supplier failure, and to gain access to a wider set of supplier capabilities (see, e.g., Slack et al. 2007). Multi-sourcing also has the potential to enable a manufacturer to encourage competition between its suppliers, on price and a variety of service-related dimensions. However, multi-sourcing may not always minimize the buyer’s operational costs, particularly in the presence of economies of scale or demand uncertainty. Thus, two key questions arise: (1) How can a buyer who has chosen to multi-source encourage competition between the suppliers on multiple dimensions, e.g., price and service? (2) Do the competitive benefits of multi-sourcing outweigh the resulting increase in operational costs? In this paper, we attempt to address these questions.

We consider a supply chain consisting of a single buyer (e.g., an OEM) who outsources the manufacturing of a given product to one or more symmetric make-to-stock suppliers. We start by assuming that the buyer has made a decision to work with multiple suppliers, for the reasons outlined above. The buyer faces stochastic demand for the product and allocates this demand between the suppliers using a score function that has an exponential form and which depends on the prices and service levels, as measured by the fill-rates, provided by the suppliers. The buyer does so with the goal of minimizing his long run average cost, which includes the purchase cost and the cost of customer backorders. The key parameter of the exponential score function captures the relative importance of price vs. service level (fill rate). The buyer’s problem is to choose this score function parameter to minimize his cost. The suppliers compete on both price and service, as determined by their base stock levels, while seeking to maximize their respective profits. Given that our model incorporates both the suppliers’ reactions to the buyer and captures competition between the suppliers, we first consider a Nash equilibrium model of competition between the suppliers, followed
by a Stackelberg game to capture the interaction between the buyer and suppliers.

For this problem setting with multi-sourcing, we seek to understand how the buyer will trade-off price and service when allocating demand between the suppliers, considering how the suppliers compete against each other to earn a share of the buyer’s demand. We also seek to understand how the buyer’s equilibrium cost, service level and price vary with the number of suppliers from which he sources. We then consider a model of single-sourcing, under the assumption that the buyer is the Stackelberg leader. We analyze the equilibrium behavior of the system and compare the results to both the multi-sourcing setting and a centralized setting, in which the buyer owns the suppliers.

The rest of this paper is organized as follows. We first review the relevant literature and discuss our key contributions. In Section 2 we present our model and analysis for the multi-sourcing case. In Section 3 we analyze a centralized system with multi-sourcing, including a discussion of coordination. In Section 4 we analyze the single-sourcing case and compare the results to those for multi-sourcing. Section 5 presents a series of numerical examples to highlight our key results. We conclude with a discussion of the key insights in Section 6.

1.1 Literature Review

We next review the relevant literature. Since we consider a single buyer who procures goods from multiple suppliers, we first briefly review the literature on inventory management for a buyer who procures inventory from multiple supply sources. See Minner (2003) for a review. This research generally considers a buyer who procures inventory from two or more suppliers, who may differ on one or more critical characteristics, such as price, lead time, reliability or service level, e.g., Whittemore and Saunders (1977), Moinzadeh and Nahmias (1988), Veeraraghavan and Scheller-Wolf (2006), Yazlali and Erhun (2009), Gerchak and Parlar (1990), Anupindi and Akella (1993), Parlar and Perry (1996), Tomlin (2006), etc. This literature takes the characteristics of the suppliers as fixed and focuses on the optimal procurement decision for the buyer. Thus, competition between the suppliers is not an issue.

The research presented in the current paper differs from the above literature in that we do not model the inventory decisions at the buyer. Instead, we assume that the buyer holds no inventory and that each arriving demand (order) is allocated to one of the competing
suppliers on the basis of their prices and service levels. That supplier will fill the demand if she has on-hand inventory. Otherwise, she will backorder the demand, with backorder costs incurred by the buyer. Thus, a key question for the buyer is how best to allocate demand to the competing suppliers. Given this setting, the most relevant stream of literature considers competition between suppliers when the buyer chooses among them based on a number of criteria, e.g., price, lead time, service level. For this type of problem, one of the key issues is how the buyer should design his allocation method, i.e., how the buyer’s demand should be divided between the suppliers, given the performance of the suppliers.

Ha et al. (2003) use an EOQ-like model to consider two suppliers who compete on price and delivery frequency (time between shipments). Given the suppliers’ announced prices and delivery frequencies, the buyer determines the fraction of demand to allocate to each supplier in order to minimize his long run average costs. The authors study the equilibrium behavior of the system under various assumptions regarding decision rights, i.e., which player has responsibility for which decisions (price and logistics). Like the current paper, this paper studies both the competition between the suppliers (modeled as a Nash equilibrium) and the equilibrium decisions of the buyer (modeled as a three stage Stackelberg game). However, the current paper differs in two key ways. First, we assume that the buyer proactively specifies his order allocation policy before the competition between the suppliers takes place and seeks to influence this competition by changing the parameter(s) of the allocation policy. Second, we assume that the suppliers choose both price and service level, whereas Ha et al. (2003) consider suppliers who choose either price or delivery frequency, but not both.

Benjaafar et al. (2007) consider a buyer who allocates demand for his product to N competing suppliers. The allocation decision is based only on service, and does not consider price. The authors study how competition can improve the service offered by the suppliers. The model considered in the current paper is similar to the make-to-stock model considered in Benjaafar et al. (2007), but with some key differences. First, we consider suppliers who choose both price and service level (fill rate). Second, while we focus on an allocation function based on an exponential score function, we incorporate the buyer’s decision problem of choosing the parameters of the allocation function in order to minimize his own costs. In contrast, Benjaafar et al. (2007) consider more general allocation functions, but assume the
buyer’s goal is to achieve the maximum service quality given fixed prices.

Cachon and Zhang (2007) consider a buyer who procures a service from two competing suppliers. The buyer allocates service jobs between the two suppliers with the goal of minimizing the average time to complete a job. The price paid by the buyer per job is assumed to be fixed. The suppliers choose their capacities with the goal of maximizing their average profits. The authors model the competition between the suppliers and study how the buyer’s allocation method affects the equilibrium capacities chosen by the suppliers. They consider both state-dependent and state-independent allocation methods. The current paper differs in that we assume the suppliers choose their service levels and prices, and that the buyer seeks to minimize his total cost. In addition, we focus only on state-independent allocation policies in which, given the characteristics of the suppliers, a fixed fraction of demand is allocated to each supplier. Cachon and Zhang (2003) extend Cachon and Zhang (2007) to also consider the buyer’s pricing decision. In contrast, in the current paper we assume it is the suppliers who set the price. While the setting considered in Cachon and Zhang (2003) is quite different than that considered in this paper, they find, as do we, that for most allocation methods, the buyer sees no benefit from sourcing from more than two suppliers when using multiple sourcing.

Note that the above papers consider the single buyer, multiple supplier setting from the perspective of the buyer. In contrast, Ozer and Raz (2011) consider such a setting from the perspective of one of the suppliers and examine the role of information in the contracting decision.

While the papers discussed above consider suppliers who compete for orders placed by a single buyer, a related stream of literature considers suppliers (retailers) who compete for a portion of the market demand. For our purposes, the most relevant paper from this stream of research is Bernstein and Federgruen (2004), who consider the equilibrium behavior of a supply chain in which multiple retailers compete for market demand on the basis of price and fill rate. They assume that the fraction of demand, i.e., market share, seen by each retailer can be modeled using an attraction model, such as the multinomial logit model, in which the attraction value of a given firm is a function of the price and fill rate offered by that firm. Our allocation function is similar to the multinomial logit attraction model considered.
in Bernstein and Federgruen (2004). Bernstein and Federgruen (2007) also consider multiple retailers who compete for market demand on the basis of price and service, but they extend the model to consider a two-echelon supply chain in which the retailers order from a common supplier. They develop pricing and backorder penalty schemes to coordinate the system.

Several other papers have considered a similar problem in which retailers (suppliers) compete on service and/or price for market share, e.g., Hall and Porteus (2000), Gans (2002), Cachon and Harker (2002), Allon and Federgruen (2007), Tsay and Agrawal (2000) and Boyaci and Gallego (2004). Similarly, numerous papers have considered retailers (suppliers) who compete on quality and price, e.g., Banker et al. (1998), Matsubayashi (2007) and Moorthy (1988). In all of these papers, the manner in which the market is allocated to the retailers as a function of their price and/or service/quality decisions is taken as exogenous, i.e., the allocation function is taken as fixed and given, not as a decision variable. In contrast, in the current paper, the parameters of the allocation function are chosen by the buyer in order to encourage an optimal level/type of competition between the suppliers.

1.2 Contributions and Summary of Results

In summary, we extend the existing literature by considering a supply chain which captures horizontal competition between the suppliers as well as vertical competition between the buyer and the suppliers. We do so under the assumption that the suppliers may choose both their service levels and their prices, i.e., the suppliers have a two dimensional strategy space, which significantly complicates the analysis. We note that few papers in the supply chain literature have considered such multi-dimensional strategy spaces (Zhao et al. 2005 is one exception). Using a two dimensional strategy space enables us to study how the suppliers trade-off the benefits of offering a lower price vs. higher level of service and to consider how the buyer can adjust the key parameter of his exponential score function to obtain the price-service level trade-off that will minimize his own costs.

For score functions having an exponential form, we characterize the equilibrium behavior of a system with multiple suppliers. For a problem with two symmetric suppliers, we show that a unique equilibrium exists and we characterize the behavior of this equilibrium as a function of key model parameters. We then extend the analysis to consider a general number
of symmetric suppliers and demonstrate that the increase in operational costs caused by the splitting of demands among a large number of suppliers outweighs any benefits from increased competition that would be achieved by sourcing from a large number of suppliers. Thus, the buyer’s expected cost is increasing in the number of suppliers. Motivated by this result, we also consider a model of single-sourcing, under the assumption that the buyer is the Stackelberg leader. We find that the buyer can achieve the lowest cost by working with a single supplier. In addition, we find that a decentralized setting with single-sourcing will have lower system cost than a centralized system with multi-sourcing. Thus, we conclude that buyers who are considering the use of multi-sourcing as a mechanism for promoting competition between their suppliers must carefully consider the impact of such multi-sourcing on their operations, and their total cost.

We also study the optimal behavior of a centralized system, in which a single decision-maker controls both the buyer and suppliers, and use the results to develop a coordinating contract for the decentralized system. We demonstrate that, under coordination, the suppliers charge higher prices and provide higher service levels than in the decentralized system.

2 Multi-Sourcing: Problem Description and Analysis

We consider a supply chain consisting of a single buyer (e.g., an OEM) who outsources the manufacturing of a given product to two competing suppliers. We use $i$ and $j$ as subscripts to denote the suppliers, for $i, j = 1, 2, i \neq j$. We assume symmetric suppliers and thus we drop the subscripts when doing so will not cause confusion. The assumption of symmetric suppliers would make sense, for example, in mature and competitive industries in which all of the participants have access to the same technology. The buyer (he) faces Poisson demand, with rate $\lambda$, for his product and each demand must be satisfied by one of the suppliers (she).

In analyzing this problem, we are interested in understanding the equilibrium behavior of the suppliers, i.e., the equilibrium prices and service levels, as well as the impact of the allocation method on the performance of the buyer, i.e., how the buyer can use the allocation method to improve his costs and the performance of the supply chain as a whole. We start, in the next subsection, by discussing our assumptions regarding the buyer’s allocation method.
Then, given this allocation method, we formulate the problems faced by the suppliers and by the buyer. We then analyze the equilibrium behavior of the system. Finally, we conclude this section with a discussion of how the results can be extended to the case of $N$ suppliers, with a focus on how the equilibrium prices, service levels and costs depend on $N$.

2.1 The Buyer’s Allocation Function

The buyer must specify an allocation policy, i.e., a method for splitting demands between the suppliers, in order to minimize his long run average cost. We assume this allocation is a function of the prices and service levels (fill rates) provided by the suppliers. Specifically, the buyer uses an exponential score function, $a(s_i, p_i) = e^{s_i - \alpha p_i}$, which depends on both the price charged by supplier $i$, $p_i$, and the service level provided by supplier $i$, $s_i$. Each arriving demand at the buyer is immediately allocated to one of two competing suppliers according to a given probability, $\beta_i$, $i = 1, 2$, where $\beta_i = \frac{a(p_i, s_i)}{a(p_i, s_i) + a(p_j, s_j)}$, $j \neq i$. Thus, the total demand seen by supplier $i$ is Poisson with rate $\beta_i \lambda$. Since demands arrive in a Poisson process, randomized allocation is required to preserve the Poisson process. As noted in Benjaafar et al. (2007), randomized allocation may be difficult to achieve, but can approximate “the behavior of a central dispatcher that attempts to adhere to a specified allocation for each supplier.”

We have assumed that the buyer uses a state-independent allocation policy, where the allocation depends only on the price and service at each supplier, and not on each supplier’s state. As noted in Cachon and Zhang (2007), such state-independent allocation policies may not be optimal for a buyer who sources from strategic suppliers. However, a state-dependent allocation policy would require the buyer to have knowledge of each supplier’s state, i.e., inventory level, at any point in time. This degree of information sharing will not necessarily hold in an outsourced supply chain such as the one considered in this paper. In addition, the scorecard functions found in practice generally depend on key performance indicators, such as service, not on the detailed state of each supplier. Finally, assuming state-independent allocation allows us to focus on the buyer’s allocation decision at a strategic level, i.e., as a function of the suppliers’ prices and levels of service, rather than at an operational level, where the focus would be on the day-to-day status of the suppliers. In other words, using the terminology of Bell and Stidham (1983), we consider a design model, not a control model.
A variety of state-independent allocation methods have been considered in the literature. Benjaafar et al. (2007) consider two approaches to allocation. In the first, the buyer allocates a proportion of demand to each supplier based on the service level offered by each supplier. In the second, the buyer uses a probabilistic mechanism to select a single supplier. In both cases they assume proportional allocation, i.e., the proportion allocated to supplier \( i \) (alternatively, the probability that supplier \( i \) is selected) can be calculated as
\[
\alpha_i = \frac{g(s_i)}{\sum_{j=1}^{N} g(s_j)},
\]
where \( N \) is the number of (potential) suppliers and \( g(\cdot) \) is a nondecreasing, concave function of the service level offered by supplier \( i, s_i \). Cachon and Zhang (2007) consider several state-independent allocation methods for make-to-order queues, including Bell-Stidham (1983) allocation, which minimizes the buyers lead time, and balanced allocation, developed by Gilbert and Weng (1998), which attempts to equalize the suppliers’ lead times given their capacities. Cachon and Zhang (2007) themselves propose the linear and proportional allocation methods. The state-dependent mechanisms considered in Cachon and Zhang (2007) include common-queue allocation for make-to-order systems, as described in Kalai et al. (1992), in which demands are assigned to a single queue and then allocated to the idle suppliers with equal probability. Common-queue allocation is most appropriate when the servers are symmetric. In systems with non-symmetric servers, a threshold allocation policy, as studied in Lin and Kumar (1984), minimizes the expected waiting time. In this policy, a demand is allocated to the “primary” supplier (usually the supplier with the fastest service time) only if that supplier has fewer than \( m \) demands in his queue, for some fixed \( m \). Finally, we note that most of this literature considers a make-to-order environment. There has been relatively little literature considering demand allocation policies in a make-to-stock environment.

In this paper, we assume the buyer allocates demand using a score function of the form
\[
a(s, p) = e^{s-\alpha p}.
\]
We choose to work with a score function of this form for several reasons. Obviously, a function of this form provides analytical tractability, which is critical given that we seek to characterize the equilibrium behavior of a system in which each player has a two-dimensional strategy space. In addition, a score function of this form allows us to capture the importance of price relative to service with a single parameter, \( \alpha \). This score function also allows for a full range of prices and service levels while remaining non-negative.
Similarly, this score function places no limit on $\alpha$, providing a great deal of flexibility in modeling the buyer’s preference for price vs. service. Also, our allocation model is similar to the market attraction models used in recent literature considering firms that compete for market share on multiple attributes, e.g., Bernstein and Federgruen (2004).

Finally, our score function was inspired by the supplier selection model studied in Benjaafar et al. (2007), who present a model for the probabilistic selection of a single supplier from a set of suppliers who compete on service. Their model is based on the multinomial logit (MNL) choice model, in which a customer must choose from a set of $N$ options with different values (utilities), where the value obtained from a given option is a function of some attributes of that option. For example, the value obtained from a particular option $i$ may be written as $v_i = \sum_{j=1}^{J} a_j X_{ij}$, where there are $J$ critical attributes, with $X_{ij}$ being the level of attribute $j$ on option $i$, and where $a_j$ represents the relative importance of attribute $j$. In addition, the value associated with option $i$ has an additive random component, denoted by $\epsilon_i$, which is assumed to follow a Gumbel distribution, and which represents variation or error in the measurement of preferences. Thus, the true value obtained from choosing option $i$ can be written as $V_i = v_i + \epsilon_i = \sum_{j=1}^{J} a_j X_{ij} + \epsilon_i$. For this model, the probability that a customer will choose option $i$ can be written as

$$P(V_i \geq V_j \text{ for all } j \neq i) = \frac{e^{v_i/\mu}}{\sum_{i=1}^{N} e^{v_i/\mu}} = \frac{e^{(\sum_{j=1}^{J} a_j X_{ij})/\mu}}{\sum_{i=1}^{N} e^{(\sum_{j=1}^{J} a_j X_{ij})/\mu}},$$

where $\mu$ is a constant associated with the Gumbel distribution.

In the Benjaafar et al. (2007) model, service is the only attribute considered. Thus, the score assigned to a given supplier is written as $V_i = g(s_i) + \epsilon_i$, where $g(s_i)$ is some increasing function of the service level, $s_i$. In addition, Benjaafar et al. (2007) argue that the random error term represents “inherent and unbiased randomness in the selection process ... [it] could denote the outcome of an opinion poll of the buyer’s purchasing managers or an outcome of an audit of the suppliers after the service levels have been announced.” Rather than using a generic function (as in Benjaafar et al. (2007)) to represent the value obtained from a supplier with given attributes, in our model we have chosen to use a linear function. In this case, we have $V_i = s_i - \alpha p_i + \epsilon_i$, and thus the probability the buyer prefers supplier $i$ takes
the form $\frac{e^{(s_i - p_i) / \mu}}{\sum_{i=1}^{N} e^{(s_i - p_i) / \mu}}$. If price and service level are scaled relative to $\mu$, this expression is identical to our allocation function.

Unlike Benjaafar et al. (2007), who consider the probabilistic selection of a single supplier, we consider the probabilistic allocation of orders across multiple suppliers. Thus, in our case, the probability that supplier $i$ provides the most value to the buyer is not used to select a single supplier, but rather to allocate the orders across the suppliers. The reasoning is as follows: For strategic reasons, the buyer has chosen to employ a multi-sourcing strategy. Thus, the question is not which supplier to use, but how much demand to allocate to each supplier. Given that there is some uncertainty about the true value of using a given supplier (as represented by $\epsilon_i$), the buyer sets the percentage of demand allocated to a given supplier equal to the probability that that supplier provides the most value to the buyer.

2.2 The Supplier’s Problem

Given the allocation policy, which the buyer announces to the suppliers, supplier $i$ chooses her selling price, $p_i$, and service level, $s_i$, in order to maximize her long run average profits, under competition with supplier $j$, $i, j = 1, 2, i \neq j$. We assume perfect information between the suppliers, i.e., each supplier knows the other supplier’s price and service level.

Each supplier operates on a make-to-stock basis, i.e., follows a base-stock policy with base-stock level, $B$, with exponential production times. We assume that each supplier scales her capacity, i.e., her production rate $\mu$, according to demand in order to achieve a target utilization, $\rho$, i.e., if a supplier sees demand rate $\lambda$, she will choose $\mu$ such that $\lambda / \mu = \rho$ for a given $\rho$. As noted by Benjaafar et al. (2007), this assumption is plausible in many situations (e.g., if capacity is flexible and capacity adjustments are inexpensive, a supplier may adjust the capacity assigned to the buyer in order to maintain a target utilization) and allows us to focus on the impact of the suppliers’ inventory decisions. Note that the supplier could potentially make three decisions: price, base-stock level, and utilization. We have chosen to focus on the first two decisions, fixing the third. The symmetric suppliers each incur a unit production cost, $w$, a cost per unit of capacity, $c$, and a unit holding cost $h$. The model and assumptions here follow the make-to-stock model considered in Benjaafar et al. (2007).

Given the base-stock level, $B$, a supplier’s expected inventory is just $E[I(B)] = B - \ldots$
\[ \frac{C}{1-\rho} \left(1 - \rho^B \right), \] while her fill rate is  

\[ s = P(I > 0) = 1 - \rho^B. \]  

Given this direct relationship between \( B \) and \( s \), we can reformulate the problem to be a function of only \( s \), i.e., the supplier’s decision variables are \( p \), the unit selling price, and \( s \), where the latter choice determines the appropriate base-stock level, \( B \). After doing so, we find that  

\[ E[I(s)] = \frac{\ln(1-s)}{\ln \rho} - \frac{\rho}{1-\rho} s. \]

We can now write the expected profit for supplier \( i \) as a function of her price, \( p_i \), and service level, \( s_i \), as  

\[ \Pi_i(p_i, s_i) = \beta_i \lambda \left( p_i - w - \frac{c}{\rho} \right) - h \left( \frac{\ln(1-s_i)}{\ln \rho} - \frac{\rho}{1-\rho} s_i \right), \]  

where, as defined above, \( \beta_i \) is the fraction of the buyer’s demand allocated to supplier \( i \), a function of the prices and service levels at both suppliers.

### 2.3 The Buyer’s Problem

Since the suppliers follow base-stock policies, when a demand arrives from the buyer, that demand may or may not be filled immediately. Any demands that cannot be filled immediately are backordered with the backorder cost, \( b \), incurred by the buyer. The assumption that the buyer bears full responsibility for the cost of any backorders is consistent with our model setting. Note that, prior to receiving an allocation of demand, the suppliers announce their price and service levels to the buyer. The buyer then chooses how to allocate demand based on these prices and service levels. Thus, as long as the suppliers meet the contracted fill rate, i.e., the fill rate they announce to the buyer and which the buyer uses in her order allocation, the supplier bears no responsibility for the backorder costs at the buyer. However, it is straightforward to extend our model and equilibrium analysis to consider a setting in which the buyer and supplier share the backorder costs\(^1\).

Assuming the buyer (eventually) fills all demands and sells the product to customers at a fixed price, the total revenue will be fixed. Therefore, we can formulate the buyer’s problem as one of cost minimization, where his cost function can be written as:

\(^1\)In this case, the equilibrium price is the same as that found in Section 2.4. However, the equilibrium service level is slightly different and, in particular, is increasing in the fraction of the backorder cost paid by the suppliers.
\[ C = \sum_{i=1,2} (\beta_i \lambda) p_i + b \sum_{i=1,2} \frac{\rho}{1 - \rho} (1 - s_i), \]  

(2)

where \( \frac{\rho}{1 - \rho} (1 - s_i) \) is the expected backorders incurred by supplier \( i \), \( \beta_i = \frac{a(p_i, s_i)}{a(p_i, s_i) + a(p_j, s_j)} \), \( j \neq i \), and \( a(p_i, s_i) = e^{s_i - \alpha p_i} \).

The buyer’s problem is to choose the score function parameter, \( \alpha \), to minimize his long run average cost, \( C \), where \( \alpha \) is a measure of the importance of price relative to service, i.e., higher \( \alpha \) implies that the buyer places greater weight on price in the allocation decision.

### 2.4 Equilibrium Analysis

As noted above, our model incorporates the suppliers’ reactions to the buyer’s choice of the allocation parameter and captures the impact of competition between the suppliers on the buyer. Therefore, in performing an equilibrium analysis for this model, we first consider a model of horizontal competition between the suppliers, followed by a Stackelberg game to capture the vertical interaction between the buyer and suppliers. In other words, for a given score function, i.e., for a given value of \( \alpha \), we first characterize the equilibrium prices and service levels for the suppliers. Then, given these results, we can determine the value of \( \alpha \) that minimizes the long-run average cost for the buyer.

#### 2.4.1 Analysis of Suppliers’ Equilibrium

In this section, we prove the existence of a unique, symmetric equilibrium for the game between the suppliers and characterize the equilibrium prices and service levels. We start by proving that, if an equilibrium exists, it must be symmetric, i.e., the suppliers choose the same prices and service levels. We then specify conditions under which a unique equilibrium will exist and provide closed form solutions for the equilibrium price and service level.

**Theorem 1** Suppose the suppliers are symmetric and that the buyer uses a score function of the form \( a(s, p) = e^{s - \alpha p} \). Then, considering competition between the suppliers on both price and service level, if an equilibrium exists, the equilibrium must be symmetric, i.e., \( p_1^* = p_2^* = p^* \) and \( s_1^* = s_2^* = s^* \). Further, if \([(-\ln \rho) \left( \frac{\lambda}{2 \alpha h} + \frac{\rho}{1 - \rho} \right)]^{-1} < 1\), a unique
equilibrium exists with
\[ p^* = \frac{2}{\alpha} + w + \frac{c}{\rho} \]
and
\[ s^* = 1 - \left[ (-\ln \rho) \left( \frac{\lambda}{2\alpha h} + \frac{\rho}{1-\rho} \right) \right]^{-1}. \]

Proofs of this and all other results can be found in the Appendix. The condition required for the existence of a unique equilibrium is needed to ensure that \( s^* \) is non-negative. We will defer discussion of this condition, and when it will hold, until the next section.

Finally, notice that \( \frac{\partial p^*}{\partial \alpha} = -\frac{2}{\alpha^2} < 0 \) and \( \frac{\partial s^*}{\partial \alpha} = \frac{1}{\ln \rho} \left( \frac{\lambda}{2\alpha h} + \frac{\rho}{1-\rho} \right)^{-2} < 0 \). Thus, as expected, increasing \( \alpha \), i.e., putting greater weight on price relative to service, causes the suppliers to decrease both the price and the service level.

### 2.4.2 Analysis of Buyer’s Equilibrium

When \( a(s, p) = e^{s-\alpha p} \) and the suppliers are symmetric, we know that a unique symmetric equilibrium exists with \( p(\alpha) = \frac{2}{\alpha} + w + \frac{c}{\rho} \) and \( s(\alpha) = 1 - \left[ (\ln \rho) \left( \frac{\lambda}{2\alpha h} + \frac{\rho}{1-\rho} \right) \right]^{-1} \). Therefore, the buyer’s cost function can be written as
\[ C(\alpha) = \lambda p(\alpha) + 2b \left( \frac{\rho}{1-\rho} \right)(1-s(\alpha)). \]

We can now characterize the optimal value of \( \alpha \) for the buyer:

**Theorem 2** Let \( g(b, h, \rho) = \sqrt{\left( \frac{e^\rho}{1-\rho} \right) \left( \frac{1}{\ln \rho} \right) \left( \frac{b}{2h} \right)} - \frac{b}{2h} \). Then

- If \( g(b, h, \rho) > 0 \), the value of \( \alpha \) that minimizes the buyer’s cost is \( \alpha^* = \frac{\lambda}{2h \cdot g(b, h, \rho)} \).
- If \( g(b, h, \rho) < 0 \), the value of \( \alpha \) that minimizes the buyer’s cost is \( \alpha^* = \infty \).

Recall that \( \alpha \) represents the relative importance of price compared to service. Thus, the condition \( g(b, h, \rho) > 0 \), which is required for \( \alpha^* \) to be finite, ensures that service matters to the buyer, i.e., that the buyer does not place infinite weight on price. Next, notice that (1) \( g(b, h, \rho) > 0 \) implies \( \frac{2b}{b} < \left( \frac{1-\rho}{\ln \rho} \right) \left( \frac{1}{\ln \rho} \right) \) and that (2) \( \left( \frac{1-\rho}{\rho} \right) \left( \frac{1}{\ln \rho} \right) \) decreases from \( \infty \) to one as \( \rho \) increases from zero to one. Thus, if \( b \geq 2h \), i.e., if the backorder cost is large enough
relative to the holding cost, the condition \( g(b, h, \rho) > 0 \) will always hold and \( \alpha^* \) will be finite, implying that the buyer will place some importance on both price\(^2\) and service.

**Corollary 1** If the following condition holds

\[
\frac{2h}{b} < \left( \frac{1 - \rho}{\rho} \right) \left( \frac{-1}{\ln \rho} \right) < \frac{b}{2h}, \tag{3}
\]

the equilibrium price and service level will be

\[
p(\alpha^*) = \frac{2}{\alpha^*} + w + \frac{c}{\rho} \quad \text{and} \quad s(\alpha^*) = 1 - \sqrt{\left( \frac{1 - \rho}{\rho} \right) \left( \frac{-1}{\ln \rho} \right) \left( \frac{2h}{b} \right)}, \tag{4}
\]

where \( 0 < s(\alpha^*) < 1 \). Finally, the equilibrium cost for the buyer will be

\[
C(\alpha^*) = \lambda p(\alpha^*) + 2b \left( \frac{\rho}{1 - \rho} \right) (1 - s(\alpha^*)).
\]

To find the equilibrium price and service level for the suppliers, we simply plug the expression for \( \alpha^* \) into \( p(\alpha) \) and \( s(\alpha) \). The expression for \( p(\alpha^*) \) does not simplify significantly, but the expression for \( s(\alpha^*) \) simplifies to (4). In order to ensure that \( s(\alpha^*) > 0 \), we require

\[
\sqrt{\left( \frac{1 - \rho}{\rho} \right) \left( \frac{-1}{\ln \rho} \right) \left( \frac{2h}{b} \right)} < 1,
\]

which implies

\[
\left( \frac{1 - \rho}{\rho} \right) \left( \frac{-1}{\ln \rho} \right) \left( \frac{2h}{b} \right) < \frac{b}{2h}.
\]

This condition, together with \( g(b, h, \rho) > 0 \), results in condition (3) in the corollary. Notice that (3) will hold for most common values of \( h, b \) and \( \rho \). As noted above, if \( h < b/2 \) the lower bound will always hold. When \( \rho \geq 0.3, \left( \frac{1 - \rho}{\rho} \right) \left( \frac{-1}{\ln \rho} \right) < 2 \). Thus, the upper bound will hold for \( b \geq 4h \). In other words, the results in this paper will generally hold as long as \( \rho \geq 0.3 \) and \( b \geq 4h \) (and may hold in other cases as well).

Finally, we have the following results on the equilibrium behavior of the system:

**Theorem 3** If \( b \geq 4h \), the following results hold:

- \( \alpha^* \) is decreasing in \( h \) and \( b \), increasing in \( \lambda \), and constant in \( c \).
- \( p(\alpha^*) \) is increasing in \( h \), \( b \) and \( c \), and decreasing in \( \lambda \).
- \( s(\alpha^*) \) is increasing in \( b \) and \( \rho \), decreasing in \( h \), and constant in \( c \) and \( \lambda \).

\(^2\)Notice that \( \alpha^* > 0 \).
The results indicate that as the backorder and holding costs increase, the buyer will place less weight on price in the score function, i.e., as these costs increase, service becomes more important to the buyer, and the price charged by the suppliers will increase. As expected, the service level seen by the buyer will increase as the backorder cost increases and decrease as the holding cost increases. In addition, the service level is increasing in the utilization. Notice that we are not able to say anything about the behavior of $\alpha^*$ and $p(\alpha^*)$ as a function of the utilization, $\rho$. This is because the function $g(b, h, \rho)$ is not strictly increasing or decreasing in $\rho$, even under the conditions in Theorem 3. However, our computational results indicate that $\alpha^*$ and $p(\alpha^*)$ will generally, although not always, be decreasing in $\rho$.

2.5 Extension to Case of $N$ Suppliers

We conclude this section by considering how the results presented above can be extended to the case of $N$ suppliers, for $N \geq 2$. We are particularly interested in understanding the impact of the number of suppliers on the buyer’s equilibrium cost and in determining whether increased competition, obtained by sourcing from many suppliers, can improve the performance of the buyer. It is analytically challenging to discuss the general equilibria. Thus, we focus on the symmetric equilibria, without drawing any conclusions regarding non-symmetric equilibria. Our results are summarized in the following theorem.

**Theorem 4** If $\frac{N-1}{N-1} \frac{b}{h} < \left(\frac{1-\rho}{\rho}\right) \left(\frac{1}{\ln \rho}\right) < \frac{N-1}{N} \frac{b}{h}$, there exists a unique, symmetric equilibrium and the equilibrium prices and service levels can be written as:

$$s(\alpha^*) = 1 - \sqrt{\left(\frac{N}{N-1}\right) \left(\frac{h}{b}\right) \left(\frac{1-\rho}{\rho}\right) \left(-\frac{1}{\ln \rho}\right)},$$

and

$$p(\alpha^*) = \left(\frac{N^2}{N-1}\right) \left(\frac{h}{\lambda}\right) \left[\left(\frac{1-\rho}{\ln \rho}\right) \left(\frac{1}{1-s(\alpha^*)}\right) - \frac{\rho}{1-\rho}\right] + w + \frac{c}{\rho}.$$

The buyer’s equilibrium $\alpha$ and cost can be written as:

$$\alpha^* = \frac{\lambda}{Nh g(N, b, h, \rho)}, \quad \text{where} \quad g(N, b, h, \rho) = \sqrt{\left(\frac{N-1}{N}\right) \left(\frac{b}{h\lambda}\right) \left(\frac{\rho}{1-\rho}\right) \left(-\frac{1}{\ln \rho}\right) - \frac{\rho}{1-\rho}}.$$
Finally, the equilibrium prices, the equilibrium service levels and the buyer’s equilibrium cost are all increasing in the number of suppliers, \( N \), while the optimal score function parameter, \( \alpha^* \), is decreasing in the number of suppliers, \( N \).

The condition required for Theorem 4 is analogous to (3) in Theorem 2, and is needed to ensure that the equilibrium \( \alpha^* \) is finite and that the equilibrium service level is positive. We also note that this condition is no stronger than (3), i.e., if the condition \( \frac{N \cdot h}{N-1} \cdot \frac{1}{\rho} \cdot \frac{1}{\ln(\rho)} < 1 \) holds for \( N = 2 \) (the requirement for Theorem 2), then it will also hold for \( N > 2 \) (the requirement for Theorem 4).

We can explain these results as follows. The buyer allocates the orders to the \( N \) suppliers in a static manner, i.e., independently of the current status of the suppliers. Thus, as the number of suppliers increases, the orders are allocated to more and more separate queues and are served by slower machines (since the utilization rate is fixed). Thus, for given \( p \) and \( s \), the buyer sees increased backorders as the number of suppliers increases. This can be easily seen by extending equation (2), the cost function for the buyer when \( N = 2 \), to consider \( N \geq 2 \). Since we consider only the symmetric equilibrium, the prices and service levels are the same for all suppliers, and the buyer’s cost function can be written as

\[
C = \sum_{i=1}^{N} (\beta_i \lambda) p_i + b \sum_{i=1}^{N} \frac{\rho}{1 - \rho} (1 - s_i) = \lambda p + Nb \frac{\rho}{1 - \rho} (1 - s).
\]

Thus, if price and service are fixed, the buyer’s backorder cost increases linearly in the number of suppliers, \( N \).

Because of this result, as the number of suppliers increases, improving service level becomes more critical. Thus, with more suppliers, the buyer would decrease \( \alpha \) in order to incent the suppliers to provide a higher level of service. In other words, the buyer would be willing to accept a higher price from the suppliers in exchange for a higher service level. Such an adjustment helps the buyer to offset some of the increase in backorders that would occur as a result of the increasing number of independent queues. However, overall, the buyer’s total cost still increases. In other words, the increase in operational costs caused
by the splitting of demands among a large number of suppliers outweighs any competitive benefits that would be achieved from sourcing from a large number of suppliers. Therefore, it is preferable for the buyer to use only two suppliers or, alternatively, to allocate the orders to the suppliers in a state-dependent manner.

Figure 1 shows the behavior of the equilibrium price and service levels for the suppliers as a function of the number of suppliers, \( N \). In the figure, we set \( \lambda = 100, b = 1, h = 0.1, w = 1, c = 10 \) and \( \rho = 0.9 \). As can be seen from the figure\(^3\), the equilibrium service level is increasing and concave in \( N \), while the equilibrium price increases approximately linearly in \( N \). Thus, while the marginal benefit, in terms of improved service level, of having an additional supplier is decreasing, the marginal cost, in terms of increased price, of having an additional supplier appears to be constant. In other words, as the number of suppliers increases, the additional benefit to the buyer from improved service diminishes, while the additional cost to the buyer from increased price remains (approximately) constant.

Finally, Theorem 4 indicates that a smaller number of suppliers is better for the buyer. However, the theorem only applies for \( N \geq 2 \). In other words, the analysis in this section assumes that the buyer has decided to multi-source. Given that decision, the questions that remain are how many suppliers to use and how to allocate demand between them. These questions are addressed in Theorem 4. In Section 4, we will discuss the single-sourcing case, and compare the results for the buyer for \( N = 1 \) and \( N \geq 2 \).

\(^3\)Since \( \lim_{N \to \infty} \frac{N}{N - 1} = 1 \), it is easy to see from the expressions for \( p(\alpha^*) \) and \( s(\alpha^*) \) in Theorem 4 that as \( N \) increases, \( s(\alpha^*) \) becomes approximately constant in \( N \), while \( p(\alpha^*) \) becomes approximately linear in \( N \).
3 Centralized Supply Chain and Coordination

In this section, we characterize the optimal behavior for the two supplier system under the assumption that the supply chain is controlled by a single decision-maker. We then compare the performance of the decentralized supply chain with that of the centralized supply chain and consider how the supply chain can be coordinated.

3.1 Centralized Supply Chain

In a centralized supply chain, the buyer will allocate demand between two alternative manufacturing facilities. Since we have symmetric facilities, we assume that the buyer will split the demand equally between the two facilities, i.e., an arriving demand is sent to facility $i$ with probability 0.5, for $i = 1, 2$. The only decision variable for the buyer is the service level (or, equivalently, the base-stock level), at the manufacturing facilities (i.e., at the suppliers). The price charged by the suppliers to the buyer is no longer relevant. Let $C_c(s)$ denote the total supply chain cost for the centralized system, which we can write as:

$$C_c(s) = \lambda \left( w + \frac{c}{\rho} \right) + 2h \left( \frac{\ln(1-s)}{\ln \rho} - \frac{\rho}{1-\rho}s \right) + 2b\frac{\rho}{1-\rho}(1-s).$$  \tag{5}$$

We can then characterize the optimal service level for the centralized system, as follows.

**Theorem 5** The optimal service level, $s^*_c$, for the centralized system is:

- If $\left( \frac{-1}{\ln \rho} \right) \left( \frac{1-\rho}{\rho} \right) \left( \frac{h}{h+b} \right) < 1$, then $s^*_c = 1 - \left[ \left( \frac{-1}{\ln \rho} \right) \left( \frac{1-\rho}{\rho} \right) \left( \frac{h}{h+b} \right) \right]$.
- If $\left( \frac{-1}{\ln \rho} \right) \left( \frac{1-\rho}{\rho} \right) \left( \frac{h}{h+b} \right) \geq 1$, then $s^*_c = 0$.
- $s^*_c$ is increasing in the utilization, $\rho$, decreasing in the holding cost, $h$, and increasing in the backorder cost, $b$.

Notice that the condition $\left( \frac{-1}{\ln \rho} \right) \left( \frac{1-\rho}{\rho} \right) \left( \frac{h}{h+b} \right) < 1$ can be rewritten as $\left( \frac{1-\rho}{\rho} \right) \left( \frac{1}{\ln \rho} \right) < \frac{h+b}{h}$.

Since $\frac{h+b}{h} > \frac{b}{2h}$, the condition in Theorem 5 for a positive service level will hold if (3) holds.

Notice that, since we assume constant utilization, i.e., for a given $\lambda$, supplier $i$ chooses $\mu_i$ so that $\frac{\lambda_i}{\mu_i} = \rho$, the results obtained in this section remain the same for any allocation of demands across the two facilities.
3.2 Coordination of the Decentralized Supply Chain

We next consider how the decentralized supply chain can be coordinated. To achieve coordination, we need to ensure that each supplier chooses her base-stock level in order to achieve a service level equal to $s^*_c$, as given in Theorem 5. Thus, we want to find $\alpha$ such that $s(\alpha) = s^*_c$. Solving this condition for $\alpha$ we have $\alpha_c = \left(\frac{\lambda}{2\pi}\right) \left(\frac{1 - \rho \rho}{\rho}\right)$.

If the buyer chooses $\alpha = \alpha_c$, then the supply chain will be coordinated. This coordination will decrease (relative to the equilibrium solution) the total supply chain cost and increase the profits of the suppliers. However, the buyer’s cost will increase. This increase in cost for the buyer could be off-set by offering a contract of the following form: Each supplier pays $K$ to participate in the contract and may set price and service level as she chooses. The buyer will allocate demand between the suppliers proportionally using the score function $a(s, p) = e^{s - \alpha_c p}$, where $\alpha_c = \left(\frac{\lambda}{2\pi}\right) \left(\frac{1 - \rho \rho}{\rho}\right)$. Then, if $K$ is chosen to satisfy (6), both the buyer and the suppliers will have an incentive to participate in the contract:

$$C(\alpha_c) - 2K \leq C(\alpha^*), \quad \Pi(p(\alpha_c), s(\alpha_c)) - K \geq 0.$$ (6)

The first condition above ensures that the buyer’s cost is no higher under the coordinating contract than in the decentralized setting, while the second condition ensures that the suppliers earn positive profits under the coordinating contract$^5$. We next state the following:

**Theorem 6** There always exists a value of $K$ satisfying condition (6).

Given that he is the Stackelberg leader, the buyer will most likely choose $K$ to minimize his own cost, i.e., we would expect the buyer to set $K = \Pi(p(\alpha_c), s(\alpha_c))$.

Finally, it is useful to compare the coordinated and decentralized results:

**Theorem 7** Let $\alpha_c$ denote the weight placed on price in the buyer’s score function under the coordinating contract. Let $\alpha^*$ denote the weight placed on price in the buyer’s score function in the decentralized system. Let $p(\alpha_c)$ and $s(\alpha_c)$ denote the price and service level chosen by

$^5$Of course, it is possible that the suppliers’ reservation profit may be something other than 0, e.g., if the suppliers’ have some outside opportunities, besides working with this buyer. In this case, the feasible region for $K$ could be adjusted appropriately.
the suppliers in the system with the coordinating contract. Let \( p(\alpha^*) \) and \( s(\alpha^*) \) denote the price and service level chosen by the suppliers in the decentralized system. Then

- \( \alpha_c \) is increasing in \( \lambda \), decreasing in \( b \) and \( \rho \), and constant in \( h \) and \( c \).
- \( p(\alpha_c) \) is increasing in \( b \) and \( c \), decreasing in \( \lambda \), constant in \( h \) and convex in \( \rho \).
- \( s(\alpha_c) \) is increasing in \( b \) and \( \rho \), decreasing in \( h \), and constant in \( c \) and \( \lambda \).

In addition, if the following condition holds

\[
\left(\frac{1 - \rho}{\rho}\right) \left(\frac{-1}{\ln \rho}\right) < 2 \left(\frac{h + b}{h}\right) \left(\frac{h + b}{b}\right),
\]

then \( \alpha_c < \alpha^* \), \( p(\alpha_c) > p(\alpha^*) \) and \( s(\alpha_c) > s(\alpha^*) \).

Theorem 7 says that the buyer’s score function will place less weight on price under the coordinating contract than in the decentralized system, leading the suppliers to charge a higher price under the coordinating contract. However, the buyer will see a higher service level from the suppliers. To understand this result, consider how the optimal service level is chosen. In the centralized system, the service level is chosen to minimize the total system cost, \( C_c(s) \), as given in (5). In the decentralized system, the suppliers choose their service levels to maximize their profits, \( \Pi(p, s) \), as given in (1). While \( C_c(s) \) explicitly includes the backorder cost, \( \Pi(p, s) \) does not. Instead, the supplier’s profit only indirectly captures the impact of backorders through the buyer’s choice of \( \alpha \). Thus, it is not surprising that the centralized system would see a higher service level than the decentralized system, requiring \( \alpha_c < \alpha^* \). Finally, since \( 2 \left(\frac{h + b}{h}\right) \left(\frac{h + b}{b}\right) > \frac{b}{2h} \), condition (7) will hold whenever (3) holds.

### 4 Comparison to Single-Sourcing \((N = 1)\)

Theorem 4 indicates that a smaller number of suppliers is better for the buyer. However, the theorem only applies for \( N \geq 2 \). To consider the case of \( N = 1 \), we must first determine how price and service level would be set. In our model with multiple suppliers, we assume that the buyer takes it as given that he sources from a specific set of \( N \) suppliers. The equilibrium prices and service levels are then the result of competition between these \( N \)
suppliers. In other words, we assume that the buyer does not use the threat of working with other suppliers, beyond these $N$ suppliers, to create competition in the system. If we seek to extend our model to the case of $N = 1$, we should maintain this condition, i.e., we should assume that the buyer has chosen a single supplier and does not consider the possibility of working with any other suppliers. In this setting, it is not clear how the price and service level are decided. In reality, the final price and service level will depend on the negotiation power of each party. Thus, we cannot simply extrapolate our model to the case with $N = 1$.

That said, there is some insight that can be obtained by considering a single-sourcing model that is similar to, but not completely analogous to, our multi-supplier model. Specifically, we can consider a Stackelberg game in which the buyer is the leader. If we assume that the buyer has multiple symmetric suppliers to choose from, and thus that the buyer is not “locked in” to any particular supplier, then the buyer can use the threat of selecting a different supplier to negotiate price and service level with the single selected supplier to the point where that supplier’s profits are just equal to 0. Thus, to find the equilibrium price and service level for this single-sourcing model, we would solve the following problem:

$$\min_{p,s} C_B(p, s)$$
$$s.t. \quad \Pi_S(p, s) = 0,$$

where $C_B(p, s) = \lambda p + b \left( \frac{\rho}{1-\rho} \right) (1 - s)$ and $\Pi_S(p, s) = \lambda \left( p - w - \frac{\varepsilon}{\rho} \right) - h \left( \frac{\ln(1-s)}{\ln \rho} - \frac{\rho}{1-\rho} s \right)$. Solving this problem, we find the equilibrium price and service level, $s^*$ and $p^*$:

$$s^* = 1 - \left( \frac{h}{h + b} \right) \left( \frac{-1}{\ln \rho} \right) \left( \frac{1 - \rho}{\rho} \right)$$
and

$$p^* = w + \frac{c}{\rho} + h \left( \frac{\ln(1-s^*)}{\ln \rho} - \frac{\rho}{1-\rho} s^* \right).$$

Notice that $s^*$ is the optimal service level for the centralized supply chain, i.e., $s^* = s^*_c$, as given in Theorem 5. In other words, in this Stackelberg setting, the decentralized system achieves the system-optimal level of service.

Given $p^*$ and $s^*$, the optimal cost for the buyer under single-sourcing can be written as

$$C^*_B = \lambda \left( w + \frac{c}{\rho} \right) + h \left( \frac{\ln(1-s^*)}{\ln \rho} - \frac{\rho}{1-\rho} s^* \right) + b \left( \frac{\rho}{1-\rho} \right) (1 - s^*).$$ (8)
Next, we compare $C^*_B$ to the optimal system cost in the centralized system with $N = 2$, $C_e(s^*_c)$, as given in equation (5) in Section 3. Since $s^* = s^*_c$, we have $C^*_B < C_e(s^*_c)$. Thus, because the supplier earns no profits in the single-sourcing system, we conclude that the buyer has lower costs in the single-sourcing system than in the (decentralized or centralized) dual-sourcing system. The intuition here is the same as for the $N$ supplier case in Section 2.5, i.e., in the single-sourcing system there are fewer separate queues with faster servers (because of the constant utilization assumption) than in the dual-sourcing system.

In summary, if we consider a model of single-sourcing in which the buyer is the Stackelberg leader, and thus is able to extract all profits from the single supplier, we find that the decentralized system achieves the system-optimal level of service and thus that single-sourcing outperforms multi-sourcing. In other words, the buyer can achieve the lowest cost by working with a single supplier. We also find that a decentralized setting with single-sourcing will have lower system cost than a centralized (or coordinated) system with multi-sourcing. We conclude that buyers who are considering the use of multi-sourcing as a mechanism for promoting competition between their suppliers must carefully consider the impact of such multi-sourcing on their operations. In certain settings, such as the one considered in this paper, the competitive benefits of multi-sourcing are not sufficient to outweigh the increase in operational costs associated with multi-sourcing. There may be other settings, however, in which these results do not apply. For example, consider a setting in which the buyer does not have complete power over the supplier and thus cannot extract all of the supplier’s profits, e.g., due to dependencies that develop over the course of the buyer-supplier relationship. In such a situation, the supplier may be able to extract a profit from the buyer and thus it is not clear whether single-sourcing would continue to outperform multi-sourcing for the buyer. In addition, we note that our finding that the buyer’s cost is increasing in the number of suppliers is due, in part, to the assumption of a state-independent allocation method.

5 Numerical Examples

In order to study the equilibrium behavior of the system, and to quantify the value of coordination for the system, we examined a large number of numerical examples, for the case
Table 1: Sensitivity Analysis for Decentralized and Centralized Solutions

<table>
<thead>
<tr>
<th>Decentralized Solution</th>
<th>Centralized Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>price</td>
<td>$\rho$</td>
</tr>
<tr>
<td>service level</td>
<td>$\rho$</td>
</tr>
</tbody>
</table>

of $N = 2$. We computed the decentralized and centralized (coordinated) solutions, i.e., price, service level, $\alpha$, supplier profits and buyer costs, using the closed form expressions provided in Sections 2 and 3. We did so for a wide variety of values for the input parameters, $h, b, c, \lambda$ and $\rho$, with $w$ normalized to $w = 1$. The ranges for the parameters are $h \in [0.1, 3.1], b \in [5, 905], c \in [10, 150], \lambda \in [1, 1501], \rho \in [0.05, 0.95]$. Below we present the typical behavior of the system, i.e., while the graphs present the results for one particular problem instance, they are representative of the system behavior for other values of the problem parameters.

Note that we have analytically demonstrated how the equilibrium solution will behave as a function of the various problem parameters. We summarize our results in Table 1, where ↑ denotes increasing, ↓ denotes decreasing, ↔ denotes constant and ? denotes undetermined.

Given these results, we are most interested in comparing the solutions for the decentralized and coordinated systems. Figures 2-4 compare the optimal $\alpha$, service levels and prices in the decentralized system with those in the coordinated system, and illustrate the magnitude of the difference between these systems. (In these three figures, the solution for the decentralized system is denoted in the legend with an “*”, while the solution for the coordinated system is denoted in the legend with a “c”.) In these figures, we set $c = 10, h = 1, b = 10$. We first plot, in Figure 2, the optimal value of of the score function parameter, $\alpha$, for both systems against the utilization rate, $\rho$, for various values of the demand rate, $\lambda$. The figure indicates that $\alpha$ is decreasing in $\rho$. Although we have shown this result analytically for the coordinated system, we were unable to prove it for the decentralized system. Finally, the figure indicates that $\alpha^* > \alpha_c$ and that the difference, $\alpha^* - \alpha_c$, decreases as $\rho$ increases and as $\lambda$ decreases. In other words, the additional weight placed on price in the decentralized system, relative to the centralized system, decreases as $\rho$ increases and as $\lambda$ decreases.

Next, in Figure 3, we compare the service levels offered by the suppliers in the decentral-
ized system with those in the coordinated system, for various values of $\rho$ and $\lambda$. As shown in Corollary 1 and Theorem 5, these equilibrium service levels are constant with respect to $\lambda$. See also Table 1. As expected, the figure shows that the service level in the decentralized system is lower than that in the coordinated system. What is interesting to note is the magnitude of the difference. For the specific instances shown in the figure, the service level for the coordinated system is 30-40% higher than that of the decentralized system.

We can explain the behavior observed in Figures 2 and 3 as follows. As $\rho$ increases, the capacity becomes more stringent and more inventory is needed to fulfill a given service level. Thus, an increase in $\rho$ causes an increase in both the holding and backorder costs. However, if the unit backorder cost is significantly higher than the unit holding cost, the backorder cost will increase at a faster rate than the holding cost. The system can then lower its total costs by increasing the service level so as to avoid excessive backorders, i.e., more weight should be placed on service level in the score function ($\alpha$ decreases) as utilization increases.

![Figure 2: Comparison of equilibrium $\alpha$ for the coordinated (c) and decentralized (*) systems, as a function of $\rho$ for various values of $\lambda$](image)

In Figure 4, we compare the price charged by the suppliers in the decentralized system with that in the coordinated system, for various values of $\rho$ and $\lambda$. The figure indicates that prices are generally decreasing in $\rho$ and $\lambda$. However, as the utilization approaches one, the price in the coordinated system may begin to increase. Finally, as shown analytically, the price is higher in the coordinated system than in the decentralized system. However, the figure indicates that this difference is generally quite small. This can be explained by noting
that, generally, as $\rho$ increases, capacity becomes more fully utilized (less expensive) and the suppliers can offer a lower price. Since $\alpha$ is lower in the coordinated system, the price offered by the suppliers is higher in the coordinated system than in the decentralized system.

We next compare the total system cost for the decentralized system with that of the centralized or coordinated system in Figure 5. In the figure, we set $\lambda = 100, h = 1, b = 10$. Let $C^*_d$ denote the optimal system cost of the decentralized system, which is computed as $C^*_d = C(\alpha^*) - 2 \times \Pi(p(\alpha^*), s(\alpha^*))$, where $\Pi$ is the profit function for each of the identical suppliers and $\alpha^*$ is the optimal value of $\alpha$ for the buyer, as given in Theorem 3. Let $C^*_c$ denote the optimal system cost of the centralized system, i.e., $C^*_c = C_c(s^*_c)$, where the optimal service level for the centralized system, $s^*_c$, is given in Theorem 5.

In Figure 5, we plot the percentage cost reduction for the coordinated system, relative to the decentralized system, i.e., $\frac{(C^*_d - C^*_c)}{C^*_c}$, against the utilization rate, $\rho$, for various values of the capacity cost, $c$. As can be seen in the figure, the percentage cost reduction is increasing in $\rho$ and decreasing in $c$, indicating that, as expected, coordination is more critical in settings with highly utilized resources. In addition, our computational results indicate that the percentage cost reduction increases in $b$ and $h$ and decreases in $\lambda$. In other words, as the utilization ($\rho$) becomes more tight, or as the cost of over- or under-stocking ($h$ or $b$) increases, careful inventory management becomes more critical and thus coordination becomes more important. On the other hand, as the capacity cost ($c$) or demand rate ($\lambda$) increase, the costs
due to over- and under-stocking become a smaller component of the total system cost, and thus coordination becomes less critical. Mathematically, notice that both the centralized and decentralized system cost functions have terms of the form \((1 - \frac{\rho}{1 - \rho})s\), representing backorder and holding costs, respectively. It is easy to show that as \(\rho\) increases, \(s(\alpha^*) - s(\alpha_c)\) decreases, i.e., the difference between the service levels for the decentralized system and coordinated systems decreases. This causes the cost difference between the coordinated and decentralized systems to decrease. However, as \(\rho\) increases, \(\frac{\rho}{1 - \rho}\) also increases, approaching infinity as \(\rho\) approaches 1. Thus the difference in the service levels becomes increasingly magnified as \(\rho\) increases, resulting in an increasing difference in the holding and backorder costs.

6 Conclusions and Managerial Insights

The use of outsourcing and contract manufacturing has increased dramatically in recent years. Given that a buyer (e.g., an OEM) has made the decision to outsource, several important questions arise, including the number of suppliers to source from and how the buyer should set the relative importance of price and service level (fill rate) when allocating demand between competing suppliers. This paper is an attempt to address these questions.

We first considered the problem faced by a buyer who has chosen to dual-source, and who
must decide how to allocate demands between the suppliers in order to foster competition on both price and service. Using an exponential score function, we studied how the buyer should weight price relative to service when making his allocation decision. We found that the weight placed on price relative to service should be decreasing in the buyer’s backorder cost, since service becomes more critical as the backorder cost increases, but increasing in the demand rate. In addition, the weight placed on price relative to service should be decreasing in the suppliers’ holding cost, since with a higher holding cost the suppliers need additional incentives to increase their base-stock level and fill rate. We then extended our model to consider $N$ suppliers and studied the impact of the number of suppliers on the buyer’s performance. We found that while service level increases as the number of suppliers increases, so does price. Overall, the buyer’s cost is increasing with the number of suppliers, suggesting that the buyer may do best to dual-source.

Motivated by the observation that the supplier does better when sourcing from a smaller number of suppliers, we then considered a model of single-sourcing in which the buyer is the Stackelberg leader, and thus is able to extract all profits from the single supplier. For this setting, we found that the decentralized system is able to achieve the system-optimal level of service and, as a result, single-sourcing outperforms multi-sourcing. Thus, we conclude that if the buyer has multiple suppliers to choose from, and is not “locked in” to any particular supplier, then the buyer can achieve the lowest expected costs by single-sourcing. We note,
however, that the model presented in this paper does not incorporate a number of other factors that may motivate a buyer to multi-source, as described in Section 1, e.g., to protect against supplier failure or to access a more diverse set of supplier capabilities.

Given our equilibrium results, we then considered the question of how the performance of a system with two independent suppliers would compare to that of an integrated system in which the buyer owns the two suppliers. We characterized the optimal solution for the integrated system and found that the buyer’s costs can be significantly higher in the decentralized system, while his service level may be substantially lower, particularly when the supplier’s utilization is high. Thus, we were motivated to consider methods for coordinating the decentralized system. We found that the weight placed on price relative to service in the score function can be used as a mechanism to achieve coordination. In particular, in the decentralized setting the buyer can set the relative weight placed on price in order to incent the suppliers to choose the system optimal service level. This results in substantially higher service levels, without a significant increase in price. Our results suggest that the buyer can improve his performance, in terms of both cost and service, by adjusting the relative importance placed on price and service when making allocation decisions to the suppliers.

Finally, we note that our results rely on the assumption of an exponential form for the score function used to allocate demands between the suppliers. Unfortunately, considering other score function forms is difficult. While it is possible to extend the results to score functions of the form $a(s, p) = (e^{s - \alpha p})^\gamma$, which allows the buyer to set two parameters, $\alpha$ and $\gamma$, and thus provides additional flexibility, we have not been able to obtain results for non-exponential score functions. In order to obtain more general results, we have considered the case of fixed service levels. In this case, the buyer allocates demand on the basis of both price and service level, but the supplier’s service levels are fixed, due perhaps to budget or space limitations. Notice that, if the fixed service levels at the suppliers differ, then their score functions will differ, i.e., supplier $i$’s score function will be $a_i(p) = a(p, s_i)$. For this problem, for general score function forms, we can show that the supplier game is supermodular, and thus that an equilibrium exists. In addition, if the service levels are not equal, i.e., if $s_1 \neq s_2$, then the equilibrium prices and allocations will not be symmetric. These results can be considered an extension of the results in Bernstein and Federgruen (2004).
7 Acknowledgements

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8 References


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Technical Appendix for:

Price and Service Competition in an Outsourced Supply Chain
9 Appendix

Proof of Theorem 1. We first present, in the form of a lemma, some results that will be used in the rest of the proof:

**Lemma 1** \(-\frac{1}{\ln \rho} > \frac{\rho}{1-p}\) for \(0 < \rho < 1\) and \(e^s < \frac{1}{1-s}\) for \(0 < s < 1\)

**Proof:** We first note that the difference \(\frac{1-e^s}{\rho} - (-\ln \rho)\) is equal to 0 at \(\rho = 1\). The derivative of this difference with respect to \(\rho\) is just \(-\frac{1}{\rho^2} + \frac{1}{\rho} < 0\), for \(0 < \rho < 1\). So \(\frac{1-e^s}{\rho} - (-\ln \rho)\) is positive and decreases to 0 as \(\rho\) increases towards 1. Thus, we have \(-\ln \rho < (1-\rho)/\rho\), and consequently, \(-1/\ln \rho > \rho/(1-\rho)\).

Similarly, the difference \(1-s - e^{-s} = 0\) when \(s = 0\). The derivative of the difference is just \(-1 + e^{-s} < 0\), for \(0 < s < 1\). So \(1-s - e^{-s}\) is negative and decreases from 0 as \(s\) increases. Thus we have \(1-s < e^{-s}\), and consequently, \(1/(1-s) > e^s\). ■

Next we will prove that if an equilibrium exists, it must be symmetric, i.e., the equilibrium prices and service levels must be the same at both suppliers. To do so, we will assume that the equilibrium is not symmetric and demonstrate that there is a contradiction. In doing so, without loss of generality, we assume \(s_1 \leq s_2\). There are then four cases to consider:

1. \(s_1 = s_2\) and \(p_1 \neq p_2\). Without loss of generality, we assume \(p_1 > p_2\).

2. \(s_1 < s_2\) and \(p_1 \geq p_2\)

   For the above two cases, we have the following from the first-order derivative

   \[
   \frac{\partial \Pi_1}{\partial s_1} - \frac{\partial \Pi_2}{\partial s_2} = \frac{e^{s_1-\alpha p_1}e^{s_2-\alpha p_2}}{(e^{s_1-\alpha p_1} + e^{s_2-\alpha p_2})^2} \lambda (p_1 - p_2) + \frac{h}{\ln \rho} \left[ \frac{1}{1-s_1} - \frac{1}{1-s_2} \right] > 0.
   \]

   The inequality follows from \(p_1 - p_2 > 0\) and \(\ln \rho < 0\). Thus, either \(\frac{\partial \Pi_1}{\partial s_1}\), or \(-\frac{\partial \Pi_2}{\partial s_2}\), or perhaps both, are positive. Note that \(\frac{\partial \Pi_1}{\partial s_1}\) represents the change in Supplier 1’s profit if she marginally increases her service level, while \(-\frac{\partial \Pi_2}{\partial s_2}\) represents the change in Supplier 2’s profit if she marginally decreases her service level. This means that at least one of the suppliers can improve her profit by deviating from the equilibrium. Thus we have the contradiction. We note here that the above argument applies to the boundaries of the decision space as well, i.e., the first order derivatives are valid at the boundaries and we always increase the lower service level and/or decrease the higher service level. The remainder of the proof also applies to the boundaries for the same reason.

3. \(s_1 < s_2\), \(p_1 < p_2\) and \(a(s_1, p_1) \geq a(s_2, p_2)\)

   For this case, we have the following from the first-order derivative

   \[
   \frac{\partial \Pi_1}{\partial p_1} - \frac{\partial \Pi_2}{\partial p_2} = -\alpha e^{s_1-\alpha p_1}e^{s_2-\alpha p_2} \lambda (p_1 - p_2) + \frac{e^{s_1-\alpha p_1} - e^{s_2-\alpha p_2}}{e^{s_1-\alpha p_1} + e^{s_2-\alpha p_2}} \lambda > 0.
   \]

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The inequality follows from \(-\alpha < 0\) and \(e^{s_1-\alpha p_1} - e^{s_2-\alpha p_2} = a(s_1, p_1) - a(s_2, p_2) \geq 0\). The contradiction can be seen using an argument similar to that used in the first two cases.

4. \(s_1 < s_2, p_1 < p_2\) and \(a(s_1, p_1) < a(s_2, p_2)\)

Demonstrating a contradiction is more involved in this case. We first show an inequality that will be used in the proof. Let \(\Delta s = s_2 - s_1\). When \(\Delta s = 0\) we have:

\[
\left( \frac{1}{\ln \rho} \frac{1}{1 - s_1 - \Delta s} + \frac{\rho}{1 - \rho} \right) \left( \frac{1}{\ln \rho} \frac{1}{1 - s_1} + \frac{\rho}{1 - \rho} \right)^{-1} - e^{\Delta s} = 0.
\]

Next, we have

\[
\frac{\partial}{\partial \Delta s} \left[ \frac{1}{\ln \rho} \frac{1}{1 - s_1 - \Delta s} + \frac{\rho}{1 - \rho} \right] \left( \frac{1}{\ln \rho} \frac{1}{1 - s_1} + \frac{\rho}{1 - \rho} \right)^{-1} - e^{\Delta s}
\]

\[
= \left( \frac{1}{\ln \rho (1 - s_1 - \Delta s)^2} \right) \left( \frac{1}{\ln \rho} \frac{1}{1 - s_1} + \frac{\rho}{1 - \rho} \right)^{-1} - e^{\Delta s}
\]

\[
> \frac{1 - s_1}{(1 - s_1 - \Delta s)^2} - e^{\Delta s}
\]

\[
> \frac{1}{1 - \Delta s} - e^{\Delta s}
\]

\[
> 0.
\]

The first and third inequalities follow from Lemma 1.

Thus, \(\left( \frac{1}{\ln \rho \frac{1}{1 - s_1 - \Delta s}} + \frac{\rho}{1 - \rho} \right) \left( \frac{1}{\ln \rho \frac{1}{1 - s_1}} + \frac{\rho}{1 - \rho} \right)^{-1} - e^{\Delta s}\) is positive and increases from 0 as \(\Delta s\) increases. So we have

\[
\left[ \frac{1}{\ln \rho} \frac{1}{1 - s_2} + \frac{\rho}{1 - \rho} \right] \left[ \frac{1}{\ln \rho} \frac{1}{1 - s_1} + \frac{\rho}{1 - \rho} \right]^{-1} > \frac{e^{s_2}}{e^{s_1}} > \frac{e^{s_2 - \alpha p_2}}{e^{s_1 - \alpha p_1}}.
\]

We next demonstrate the contradiction. For this case, we have the following:

\[
\frac{\partial \Pi}{\partial p_1} + \alpha \frac{\partial \Pi}{\partial s_1} = \frac{e^{s_1-\alpha p_1}}{e^{s_1-\alpha p_1} + e^{s_2-\alpha p_2}} \lambda + h \alpha \left[ \frac{1}{\ln \rho \frac{1}{1 - s_1}} + \frac{\rho}{1 - \rho} \right]
\]

\[
\frac{\partial \Pi}{\partial p_2} + \alpha \frac{\partial \Pi}{\partial s_2} = \frac{e^{s_2-\alpha p_2}}{e^{s_1-\alpha p_1} + e^{s_2-\alpha p_2}} \lambda + h \alpha \left[ \frac{1}{\ln \rho \frac{1}{1 - s_2}} + \frac{\rho}{1 - \rho} \right]
\]

\[
= \frac{e^{s_1-\alpha p_1}}{e^{s_1-\alpha p_1} + e^{s_2-\alpha p_2}} \lambda e^{s_2-\alpha p_2} + h \alpha \left[ \frac{1}{\ln \rho \frac{1}{1 - s_1}} + \frac{\rho}{1 - \rho} \right]
\]
\[
\begin{align*}
\cdot & \left[ 1 \ln \rho \frac{1}{1 - s_2} + \frac{\rho}{1 - \rho} \right] \left[ 1 \ln \rho \frac{1}{1 - s_1} + \frac{\rho}{1 - \rho} \right]^{-1} \\
& < \left( \frac{\partial \Pi_1}{\partial p_1} + \alpha \frac{\partial \Pi_1}{\partial s_1} \right) \frac{e^{s_2 - \alpha p_2}}{e^{s_1 - \alpha p_1}}
\end{align*}
\]

The inequality follows from (9). In other words, we have

\[
- \left( \frac{\partial \Pi_2}{\partial p_2} + \alpha \frac{\partial \Pi_2}{\partial s_2} \right) > \left( \frac{\partial \Pi_1}{\partial p_1} + \alpha \frac{\partial \Pi_1}{\partial s_1} \right) \left( - \frac{e^{s_2 - \alpha p_2}}{e^{s_1 - \alpha p_1}} \right)
\]

Therefore, either \(- \left( \frac{\partial \Pi_2}{\partial p_2} + \alpha \frac{\partial \Pi_2}{\partial s_2} \right)\), or \(\left( \frac{\partial \Pi_1}{\partial p_1} + \alpha \frac{\partial \Pi_1}{\partial s_1} \right)\), or perhaps both, are positive. Here \(\left( \frac{\partial \Pi_1}{\partial p_1} + \alpha \frac{\partial \Pi_1}{\partial s_1} \right)\) represents the change in Supplier 1’s profit if she increases both price and service level marginally and keeps the ratio of increase at 1 : \(\alpha\), i.e., she marginally increases the price by \(\delta\) and increases the service level by \(\alpha \cdot \delta\). Also, \(- \left( \frac{\partial \Pi_2}{\partial p_2} + \alpha \frac{\partial \Pi_2}{\partial s_2} \right)\) represents the change in Supplier 2’s profit if she decreases both price and service level marginally and keeps the ratio of decrease at 1 : \(\alpha\). This means that at least one of the suppliers can improve her profit by deviating from the equilibrium. The contradiction thus follows.

In summary, for each case in which the equilibrium is not symmetric, we have found a contradiction. Thus, if an equilibrium exists, it must be symmetric.

We next prove that a unique symmetric equilibrium always exists. We first prove the result for the case in which the equilibrium is an interior point in the decision space. At the end of the proof, we will show that the symmetric equilibrium cannot be at the boundaries. We start by writing the first order conditions that must be satisfied at equilibrium:

\[
\frac{\partial \Pi_i}{\partial s_i} = \frac{e^{s_i - \alpha p_i} e^{s_j - \alpha p_j}}{(e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j})^2} \lambda [p_i - w - c/\rho] + h \left[ 1 \ln \rho \frac{1}{1 - s_i} + \frac{\rho}{1 - \rho} \right] = 0
\]

and

\[
\frac{\partial \Pi_i}{\partial p_i} = -\alpha e^{s_i - \alpha p_i} e^{s_j - \alpha p_j} \lambda [p_i - w - c/\rho] + \frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j}} \lambda = 0,
\]

for \(i, j = 1, 2, i \neq j\). We know that, if there exists an equilibrium, it must be a symmetric equilibrium. Thus, we can use \(s_i = s_j = s\) and \(p_i = p_j = p\) to simplify the first order conditions:

\[
\frac{\partial \Pi}{\partial s} = \frac{1}{4} \lambda [p - w - c/\rho] + h \left[ 1 \ln \rho \frac{1}{1 - s} + \frac{\rho}{1 - \rho} \right] = 0
\]

\[
\frac{\partial \Pi}{\partial p} = -\frac{1}{4} \alpha \lambda [p - w - c/\rho] + \frac{1}{2} \lambda = 0.
\]
Solving for $p$ and $s$, we obtain

$$p = \frac{2}{\alpha} + w + \frac{c}{\rho} \text{ and } s = 1 + \left(\ln \rho \left( \frac{\lambda}{2\alpha h} + \frac{\rho}{1 - \rho} \right) \right)^{-1}. \quad (10)$$

Next, we show that any solution to the first order conditions is a local maximizer by demonstrating that the profit function for supplier $i$ is jointly concave around that solution. To do so, we first evaluate the second order derivatives of $\Pi_i$ w.r.t. $p_i$ and $s_i$ at an arbitrary solution to the first order conditions:

\[
\frac{\partial^2 \Pi_i}{\partial s_i^2} = \left[ e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} + e^{s_i-\alpha p_i} (-2)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-3} e^{s_i-\alpha p_i} \right] \\
\cdot e^{s_i-\alpha p_i} \lambda [p_i - w - c/\rho] + h \frac{1}{\ln \rho (1 - s_i)^2}
\]

\[
< \left[ e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} \right] e^{s_j-\alpha p_j} \lambda [p_i - w - c/\rho] + h \frac{1}{\ln \rho (1 - s_i)^2}
\]

\[
= -h \left[ \frac{1}{\ln \rho (1 - s_i)} + \frac{\rho}{1 - \rho} \right] + h \frac{1}{\ln \rho (1 - s_i)^2}
\]

\[
< h \frac{1}{\ln \rho} \left[ \frac{1}{(1 - s_i)^2} - \frac{1}{1 - s_i} \right] < 0,
\]

where the second equality is derived using the first order condition for $s_i$. In addition,

\[
\frac{\partial^2 \Pi_i}{\partial p_i^2} = \left[ (-\alpha) e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} + e^{s_i-\alpha p_i} (-2)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-3}(-\alpha) e^{s_i-\alpha p_i} \right] \\
\cdot (-\alpha) e^{s_j-\alpha p_j} \lambda [p_i - w - c/\rho] + -\alpha e^{s_i-\alpha p_i} e^{s_j-\alpha p_j} \lambda \\
+ \left[ (-\alpha) e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-1} + e^{s_i-\alpha p_i} (-1)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2}(-\alpha) e^{s_i-\alpha p_i} \right] \lambda
\]

\[
= \left[ (-\alpha) e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} + e^{s_i-\alpha p_i} (-2)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-3}(-\alpha) e^{s_i-\alpha p_i} \right] \\
\cdot (-\lambda) (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j}) + -\alpha e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j}) \lambda \\
+ \left[ (-\alpha) e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-1} + e^{s_i-\alpha p_i} (-1)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2}(-\alpha) e^{s_i-\alpha p_i} \right] \lambda
\]

\[
= -\alpha e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j}) \lambda = -\alpha e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j}) \lambda < 0,
\]

where the second equality is derived using the first order condition for $p_i$, and

\[
\frac{\partial^2 \Pi_i}{\partial s_i \partial p_i} = \left[ (-\alpha) e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} + e^{s_i-\alpha p_i} (-2)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-3}(-\alpha) e^{s_i-\alpha p_i} \right]
\]

4
\[ \dot{e}^{s_j-\alpha p_j} \lambda [p_i - w - c/\rho] + \frac{e^{s_i-\alpha p_i} e^{s_j-\alpha p_j}}{(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^2} \lambda \]

\[ = \left[ e^{s_i-\alpha p_i} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} + e^{s_i-\alpha p_i} (-2)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-3} e^{s_i-\alpha p_i} \right] \]

\[ \cdot (-\lambda)(e^{s_j-\alpha p_j} + e^{s_j-\alpha p_j}) + \frac{e^{s_i-\alpha p_i} e^{s_j-\alpha p_j}}{(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^2} \lambda \]

\[ = - \frac{e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j}}{(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^2} \lambda + \frac{e^{s_i-\alpha p_i} (2e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})}{(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^2} \lambda \]

where the second equality is derived using the first order condition for \( p_i \).

For joint concavity, we need \( \frac{\partial^2 \Pi_i}{\partial s_i^2} \frac{\partial^2 \Pi_i}{\partial p_i^2} - \left( \frac{\partial^2 \Pi_i}{\partial s_i \partial p_i} \right)^2 > 0 \). To evaluate this condition, we note that

\[ \frac{\partial^2 \Pi_i}{\partial s_i^2} \frac{\partial^2 \Pi_i}{\partial p_i^2} - \left( \frac{\partial^2 \Pi_i}{\partial s_i \partial p_i} \right)^2 = \left[ \frac{\lambda}{2} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} + e^{s_i-\alpha p_i} (-2)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-3} e^{s_i-\alpha p_i} \right] \]

\[ \cdot \frac{1}{\ln \rho (1 - s_i)^2} \]

where the second equality is derived using the first order condition for \( p_i \). Thus, for joint concavity, we need the following value to be positive:

\[ \frac{\partial^2 \Pi_i}{\partial s_i^2} \frac{\partial^2 \Pi_i}{\partial p_i^2} - \left( \frac{\partial^2 \Pi_i}{\partial s_i \partial p_i} \right)^2 = \left( \frac{\lambda}{2} (e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-2} + e^{s_i-\alpha p_i} (-2)(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^{-3} e^{s_i-\alpha p_i} \right) \]

\[ \cdot \frac{1}{\ln \rho (1 - s_i)^2} \]

\[ = \frac{\lambda}{2} \frac{1}{\ln \rho (1 - s_i)^2} \left[ \frac{\lambda}{2} - \frac{2(e^{s_i-\alpha p_i})^2}{(e^{s_i-\alpha p_i} + e^{s_j-\alpha p_j})^2} \right] \lambda \]
\[
\frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j}} \lambda \left(-\alpha h \frac{1}{\ln \rho (1 - s_i)^2} + \left[1 - \frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j}}\right]^2 \frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j}} \lambda\right)
\]

\[
> \frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j}} \lambda \left(-\alpha h \frac{1}{\ln \rho (1 - s_i)^2} - \frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j}} \lambda\right)
\]

where the last equality follows from the following combination of the first order conditions:

\[
\alpha \frac{\partial \Pi_i}{\partial s_i} + \frac{\partial \Pi_i}{\partial p_i} = \frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_j - \alpha p_j}} \lambda + \alpha h \left[1 - \frac{1}{\ln \rho (1 - s_i) + \frac{\rho}{1 - \rho}}\right] = 0.
\]

Thus, the condition for joint concavity holds. That is, for any solution to the first order conditions, the profit function is jointly concave on \(s_i\) and \(p_i\). In other words, the profit function of supplier \(i\) is a unique maximizer to supplier \(i\)’s profit maximization problem.

Next, we show that the symmetric equilibrium cannot be at the boundaries. When \(p_i = p_j \in [0, \frac{2}{\alpha} + w + \frac{\varepsilon}{\rho}]\) and \(s_i = s_j\), \(\frac{\partial \Pi_i}{\partial p_i}\) is positive. In other words, supplier \(i\) can increase his profit by increasing his price. Similarly, \(\frac{\partial \Pi_i}{\partial s_i}\) is negative and supplier \(i\) can increase his profit by decreasing his price when \(p_i = p_j > \frac{2}{\alpha} + w + \frac{\varepsilon}{\rho}\) and \(s_i = s_j\). So the symmetric equilibrium cannot be at the boundary \(p_j = p_i = 0\) and will only be possible when \(p_j = p_i = \frac{2}{\alpha} + w + \frac{\varepsilon}{\rho}\).

When \(p_i = p_j = \frac{2}{\alpha} + w + \frac{\varepsilon}{\rho}\), \(\frac{\partial \Pi_i}{\partial s_i} = 0\) at \(s_i = s_j = 1 + \left[(\ln \rho) \left(\lambda + \frac{\rho}{1 - \rho}\right)\right]^{-1}\). It is easy to see that \(\frac{\partial \Pi_i}{\partial s_i}\) is positive when \(s_i = s_j = 0\). In other words, supplier \(i\) can increase his profit by increasing his service level. Similarly, \(\frac{\partial \Pi_i}{\partial p_i}\) is negative and supplier \(i\) can increase his profit by decreasing his service level if \(s_i = s_j = 1\). So the symmetric equilibrium cannot be at boundaries \(s_i = s_j = 0\) or \(s_i = s_j = 1\).

**Proof of Theorem 2.** We would like to find \(\alpha\) to minimize the buyer’s cost. To do so, we first write the first order conditions:

\[
\frac{\partial C}{\partial \alpha} = \lambda \frac{\partial p_i}{\partial \alpha} - 2b \frac{\rho^2}{1 - \rho} \frac{\partial p_i}{\partial \alpha}, \quad \text{where} \quad \frac{\partial p_i}{\partial \alpha} = \frac{2\lambda}{\alpha^2} < 0
\]

and \(\frac{\partial s_i}{\partial \alpha} = \frac{1}{\ln \rho} \left(\frac{\lambda}{2\lambda h} + \frac{\rho}{1 - \rho}\right)^{-2} < 0\). Solving the first order condition for \(\alpha\) we find:

\[
\alpha^* = \frac{1}{2h} g(b, h, \rho), \quad \text{where} \quad g(b, h, \rho) = \sqrt{\left(\frac{\rho}{1 - \rho}\right) \left(\frac{n}{h}\right) - \frac{\rho}{1 - \rho}}.
\]

Next, we need to check to see whether \(\alpha^*\) minimizes \(C(\alpha)\). It is easy to show that if \(g(b, h, \rho) < 0\) then \(C(\alpha)\) is strictly decreasing in \(\alpha\) (\(\frac{\partial C}{\partial \alpha} < 0\)) for all \(\alpha > 0\). Thus the cost minimizing value of \(\alpha\) is \(\alpha = \infty\). Otherwise, if \(g(b, h, \rho) > 0\), it is easy to show that \(C(\alpha)\) is a decreasing function of \(\alpha\) (\(\frac{\partial C}{\partial \alpha} < 0\)) for \(\alpha < \alpha^*\) and \(C(\alpha)\) is a decreasing function of \(\alpha\) (\(\frac{\partial C}{\partial \alpha} > 0\)) for \(\alpha > \alpha^*\). Thus, if \(g(b, h, \rho) > 0\), \(\alpha^*\) is the unique optimal solution.

**Proof of Theorem 3.** Most of the results are straightforward, and thus we omit the proofs. The condition \(b \geq 4h\) is required to ensure that \(\frac{\partial |g(b, h, \rho)|}{\partial h} \geq 0\), and thus that \(\frac{\partial \alpha^*}{\partial h} \leq 0\).

**Proof of Theorem 4.** For the case of \(N\) suppliers, the profit maximization problem
for supplier \( i, i = 1, \ldots, N \), can be written as

\[
\max_{s_i, p_i} \Pi_i = \frac{e^{s_i - \alpha p_i}}{e^{s_i - \alpha p_i} + e^{s_\neq i - \alpha p_\neq i}} \lambda (p_i - w - c/\rho) - h \left[ \ln(1 - s_i) - \frac{\rho}{1 - \rho} s_i \right],
\]

where \( e^{s_\neq i - \alpha p_\neq i} = \sum_{j: j \neq i} e^{s_j - \alpha p_j} \). Given that we only consider symmetric equilibria, for a given \( \alpha \), we can derive the price and service level that satisfies the first-order conditions. Doing so, we obtain:

\[
p(\alpha) = \frac{N}{\alpha(N - 1)} + w + c/\rho
\]

\[
s(\alpha) = 1 + \frac{1}{\ln \rho} \left[ \frac{\lambda}{N \alpha h} + \frac{\rho}{1 - \rho} \right]^{-1}.
\]

Next we need to show that this solution is indeed an equilibrium by showing that the profit function is jointly quasi-concave in supplier \( i \)'s price and service level. We note here that this can be done the same way as for the case with two suppliers in the proof for Theorem 1. The uniqueness comes from the fact that this solution is the only solution to the first-order condition, given we only consider symmetric equilibria. We can then plug this solution into the buyer’s cost function and solve the buyer’s cost minimization problem to determine the optimal value of \( \alpha \). In other words, we solve the following problem:

\[
\min_{\alpha} C = \lambda p(\alpha) + N b \frac{\rho}{1 - \rho} (1 - s(\alpha))
\]

\[
= \lambda \left( \frac{N}{\alpha(N - 1)} + w + c/\rho \right) + N b \left( \frac{\rho}{1 - \rho} \right) \left( -\frac{1}{\ln \rho} \right) \left[ \frac{\lambda}{N \alpha h} + \frac{\rho}{1 - \rho} \right]^{-1},
\]

to obtain:

\[
\alpha^* = \frac{\lambda}{Nh g(N, b, h, \rho)}, \quad \text{where} \quad g(N, b, h, \rho) = \sqrt{\frac{(N - 1)b}{Nh}} \left( \frac{\rho}{1 - \rho} \right) \left( -\frac{1}{\ln \rho} \right) - \frac{\rho}{1 - \rho}.
\]

From this expression, it is easy to see that \( \alpha^* \) is decreasing in the number of suppliers, \( N \). In other words, as the number of suppliers increases, the buyer will place less emphasis on price in the score function. Next, we can plug this expression for \( \alpha^* \) into the equation for \( s(\alpha) \) to obtain:

\[
s(\alpha^*) = 1 - \sqrt{\left( 1 + \frac{1}{N - 1} \right) \left( \frac{h}{b} \right) \left( \frac{1 - \rho}{\rho} \right) \left( -\frac{1}{\ln \rho} \right)},
\]

which is clearly increasing in the number of suppliers, \( N \). Using supplier’s the first order condition for the service level, we can write the optimal price as follows

\[
p(\alpha^*) = \left( \frac{N^2}{N - 1} \right) \left( \frac{h}{\lambda} \right) \left[ \left( -\frac{1}{\ln \rho} \right) \left( \frac{1}{1 - \rho} \right) \left( s(\alpha^*) \right) - \frac{\rho}{1 - \rho} \right] + w + c/\rho
\]
Since \( \frac{1}{1-s(\alpha)} \) is increasing in \( N \), it is easy to see that the optimal price is also increasing in the number of suppliers, \( N \). Finally, plugging the price and service level into the buyer’s cost, we obtain the following expression for the buyer’s equilibrium cost:

\[
C = \lambda p(\alpha^*) + Nb \left( \frac{\rho}{1-\rho} \right) \sqrt{\left( \frac{1}{N-1} \right) \left( \frac{h}{b} \right) \left( \frac{1-\rho}{\rho} \right) \left( -\frac{1}{\ln \rho} \right)}
\]

Since \( \frac{1}{1-s(\alpha)} \) is increasing in \( N \), it is easy to see that the optimal price is also increasing in the number of suppliers, \( N \). Finally, plugging the price and service level into the buyer’s cost, we obtain the following expression for the buyer’s equilibrium cost:

\[
C = \lambda p(\alpha^*) + Nb \left( \frac{\rho}{1-\rho} \right) \sqrt{\left( \frac{1}{N-1} \right) \left( \frac{h}{b} \right) \left( \frac{1-\rho}{\rho} \right) \left( -\frac{1}{\ln \rho} \right)}
\]

The buyer’s total cost is thus increasing in the number of suppliers \( N \) as well. ■

**Proof of Theorem 5.** It is easy to verify that the second derivative of \( C_c \) with respect to \( s \) is positive and thus the centralized cost function is convex. The result then follows directly from the first order condition. ■

**Proof of Theorem 6.** Let \( C_d^i = C(\alpha_c) - 2\Pi_i^c \) denote the total system cost for the decentralized system under the coordinating contract, where \( \Pi_i^c = \Pi_i(p(\alpha_c), s(\alpha_c)) \) is supplier \( i \)'s profit in the decentralized system with the coordinating contract (excluding the fixed payment \( K \)). Let \( C_d = C(\alpha^*) - 2\Pi_i^d \) denote the total system cost for the decentralized system without the coordinating contract where \( \Pi_i^d = \Pi_i(p(\alpha^*), s(\alpha^*)) \) is supplier \( i \)'s profit in the decentralized system without the coordinating contract (excluding the fixed payment \( K \)). In each case, the total system cost is calculated as the costs at the buyer minus the sum of the (symmetric) profits at the two suppliers. Note that, under the coordinating contract, \( K \) is a transfer payment that does not affect the total supply chain cost.

Since \( \alpha^* \) is the value of \( \alpha \) that minimizes \( C(\alpha) \), \( C(\alpha_c) \geq C(\alpha^*) \). Since \( \alpha_c \) minimizes the total system cost, \( C_d^c = C(\alpha_c) - 2\Pi_i^c \leq C_d^* = C(\alpha^*) - 2\Pi_i^d \). Thus, \( C(\alpha_c) \geq C(\alpha^*) \) implies that each of the symmetric suppliers must see an increase in her profits when the buyer uses \( \alpha = \alpha_c \) rather than \( \alpha = \alpha^* \), i.e., \( \Pi_i^c \geq \Pi_i^d \). Condition (6) implies that \( K \) must satisfy \( \frac{1}{2}[C(\alpha_c) - C(\alpha^*)] \leq K < \Pi_i^c \). To prove that such a \( K \) exists we just need to show that \( C(\alpha_c) - C(\alpha^*) \leq 2\Pi_i^c \), which follows from the fact that \( C(\alpha^*) - 2\Pi_i^d \geq C(\alpha_c) - 2\Pi_i^c \). ■

**Proof of Theorem 7.** Plugging in \( \alpha_c = \left( \frac{N}{2b} \right) \left( \frac{1-\rho}{\rho} \right) \) and \( \alpha^* = \frac{\lambda}{2b} \), it is easy to show that \( \alpha_c \leq \alpha^* \) if and only if \( \left( \frac{1-\rho}{\rho} \right) \left( \frac{1}{N\rho} \right) < 2 \left( \frac{h+b}{b} \right) \). Next, we compare the prices. Notice that \( p(\alpha_c) - p(\alpha^*) = 2 \left( \frac{1}{\alpha_c} - \frac{1}{\alpha^*} \right) \). Thus, \( p(\alpha_c) > p(\alpha^*) \) if and only if \( \alpha_c \leq \alpha^* \). Finally, we compare the service levels. By definition, \( s(\alpha_c) = s_c^* \). Thus, from Theorem 5, if \( \left( \frac{1}{N\rho} \right) \left( \frac{1-\rho}{\rho} \right) \left( \frac{h+b}{b} \right) < 1 \), then \( s(\alpha_c) = 1 - \left( \frac{1}{N\rho} \right) \left( \frac{1-\rho}{\rho} \right) \left( \frac{h+b}{b} \right) \). From (4), we have \( s(\alpha^*) = 1 - \sqrt{\left( \frac{1-\rho}{\rho} \right) \left( \frac{1}{N\rho} \right) \left( \frac{h+b}{b} \right)} \). Plugging into \( s(\alpha_c) > s(\alpha^*) \) and simplifying gives the result. ■